
**Protoplanetary Disks with Misaligned Inner Disks:
Multi-Wavelength Observations
and Shadow-Based Disk Geometry Estimation**

Supervisor:
Prof. Munetake MOMOSE

Submitted January 2025

A DISSERTATION SUBMITTED TO THE GRADUATE SCHOOL OF
SCIENCE AND ENGINEERING FOR THE DEGREE OF DOCTOR
OF SCIENCE

IBARAKI UNIVERSITY

Ryuta ORIHARA

Acknowledgement

I would like to express my heartfelt gratitude to my supervisor, Professor Munetake Momose, whose unwavering support and invaluable guidance have been pivotal throughout my doctoral studies. This thesis would not have been possible without his dedicated mentorship, insightful advice, and constant encouragement at every stage of my research journey. I am particularly grateful for the many in-depth discussions we had—ranging from theoretical aspects and observational proposals to data interpretation—which greatly enriched my understanding and deepened my appreciation for the field.

I am deeply grateful to Associate Professor Takayuki Muto, the principal investigator of the observation project (2018.1.00689S) for six transitional disks, including SY Cha. The dataset he provided marked a significant milestone in my academic journey, becoming the subject of my first research project. Analyzing this dataset was both intellectually stimulating and transformative, allowing me to acquire and refine essential skills in interferometric data analysis. I sincerely appreciate his guidance and support throughout this experience.

I am also grateful to all co-authors of Orihara et al. (2023) for their valuable advice and support during the preparation of the manuscript. In particular, I would like to thank Haoyu Baobab Liu and Ruobing Dong for their insightful comments on ALMA data calibration, imaging, and interpretation of the analysis results.

My sincere thanks go to Professor Satoshi Okuzumi for his deep interest in my shadow analysis method and for suggesting a new research direction based on its application. This suggestion not only opened up a new perspective for my research but also provided an excellent opportunity to further develop my skills. I am also thankful for the time he generously shared with me during my visit to his lab, offering valuable feedback and warm encouragement despite his busy schedule.

I am indebted to Assistant Professor Masataka Aizawa for his expert advice on implementing optimization algorithms. His practical and detailed guidance served as a critical roadmap in overcoming technical challenges. Thanks to his support, I was able to significantly improve both the performance of my programs and my understanding of optimization methods.

I am also grateful to Researcher Kazuhiro Kanagawa, who provided thorough instruction on the theory of planet–disk coevolution. His insights helped me clarify the direction of my research and determine the most appropriate types of observations to pursue.

Lastly, I would like to thank Professor Yoshinori Yonekura for teaching me the technical foundations and operational principles of radio telescopes. Through his lectures and discussions, I gained a deeper understanding of radio telescope instrumentation, which greatly informed the observational aspects of this dissertation.

Abstract

This study investigates transition disks with inner disks in their innermost regions, focusing on features arising from inner disk misalignment through the analysis of optical, infrared, and millimeter-wave data. Protoplanetary disks, composed of gas and dust surrounding young stars, are thought to evolve as dust particles coagulate into kilometer-sized planetesimals, which eventually form planets. Understanding the structure and evolution of these disks is crucial for unraveling the mechanisms of planet formation. Transition disks, characterized by a significant lack of near-infrared excess in their spectral energy distribution (SED), were traditionally interpreted as having depleted dust in their inner regions. However, recent imaging observations have revealed the presence of inner disks within these cavities, necessitating a reevaluation of this interpretation.

Detecting thermal dust emission from the inner disk is crucial for characterizing its properties. However, resolving the inner disk itself remains a challenge given current observational limitations. Nevertheless, surrounding structures are believed to be strongly influenced by the presence and characteristics of the inner disk. In this study, we investigate these structures through multi-wavelength observations, aiming to clarify the inner disk's configuration and its impact on the surrounding environment.

In Chapter 2, the transition disk associated with SY Cha is analyzed as a case study of a disk lacking near-infrared excess, utilizing high-resolution observational data from the Atacama Large Millimeter/submillimeter Array (ALMA). In the innermost region of the SY Cha disk, compact thermal emission from dust in the inner disk was identified, along with a dust ring at a radius of approximately 100 au. By employing the Markov Chain Monte Carlo (MCMC) method for axisymmetric disk modeling and subtracting the resulting model from the data, non-axisymmetric structures within the dust ring were revealed. Residual maps showed a quadrupole pattern, indicative of a tilted inner disk relative to the outer disk, which led to shadowing effects and localized temperature reductions. Additionally, velocity distortions near the central star, observed in the moment 1 map of CO ($J = 2 - 1$) emission, were consistent with warped structures caused by the tilted inner disk. These findings highlight the significance of inner disk structure as a key site for terrestrial planet formation, while also demonstrating how inner disk misalignment influences the evolution of the outer disk.

In Chapter 3, a novel methodology is proposed for estimating the geometric configuration of inner disks by analyzing shadows observed in optical and infrared scattered light images. This method models the boundary between light and shadow cast by the inner disk onto the surface of the outer disk, enabling the estimation of key parameters such as the center, tilt, and aspect ratio of the inner disk, as well as the center, inner edge radius, and surface height distribution of the outer disk. The aspect ratio of the inner disk, defined as the maximum surface height divided by the radius, is particularly significant for understanding disk dynamics. Incorporating the horizon on the outer disk surface markedly improves the accuracy of the outer disk parameter estimates.

This methodology was applied to optical and infrared scattered light images of HD 100453 and HD 142527, obtained using the Very Large Telescope (VLT)/SPHERE. The analysis revealed that the inner disks of both systems are misaligned by over 50° , likely due to gravitational interactions with companion stars, and exhibit aspect ratios of approximately 0.15. For HD 142527, a localized temperature drop in the northern region, derived from submillimeter wave observations, suggested a dust cooling time much shorter than the disk rotation period, potentially triggering fluid instabilities such as the vertical shear instability.

This study demonstrates the efficacy of a novel approach for indirectly characterizing the physical and geometric properties of inner disks that are otherwise challenging to observe directly.

Contents

Acknowledgement	i
Abstract	ii
1 Introduction	1
1.1 Overview of Protoplanetary Disks	1
1.1.1 Star Formation and Evolutionary Classification of YSOs	1
1.1.2 Transition Disks	4
1.1.3 Structure of the Protoplanetary Disk	6
1.1.4 Evolution of the Protoplanetary Disk	9
1.2 Observations of Protoplanetary disks	13
1.2.1 (Sub)millimeter Observations with ALMA	14
1.2.2 Optical and Infrared Observations	16
1.3 Previous Studies on Inner Disks	19
1.3.1 Snow Line	19
1.3.2 Dead Zone Inner Edge	19
1.3.3 Misaligned Inner Disks	21
1.4 Aims to This Thesis	24
2 ALMA Band 6 high-resolution observations of the transition disk around SY Cha	25
2.1 Introduction	26
2.2 Observations and data reduction	30
2.3 Data analysis and results	31
2.3.1 Synthesized images	31
2.3.2 Analysis of dust continuum emission in visibility domain	38
2.3.3 Physical properties of the disk	42
2.4 Discussion	44
2.4.1 Radial flow	45
2.4.2 Warp	47
2.5 Conclusions	48
Appendix 2.1: Channel maps	50
Appendix 2.2: Calculation of molecular column density	53
Appendix 2.3: Corner plots of MCMC fitting results	54
Appendix 2.4 : ALMA Pipeline Calibration	55
Appendix 2.5 : Visibility weighting	57
Appendix 2.6 : CLEAN	58

Appendix 2.7 : Self-Calibration	63
Appendix 2.8 : Dust Continuum Maps	65
Appendix 2.9 : Visibility analysis of axisymmetric disks	66
Appendix 2.10 : Model Fitting Method	68
3 Shadow-Based Framework for Estimating Transition Disk Geometries	73
3.1 Introduction	73
3.2 Methods	75
3.2.1 Horizon of the Outer Disk Surface	76
3.2.2 Shadow Boundary between the Inner and Outer Disks	78
3.2.3 Gradients of Infrared Scattered Light Image	80
3.2.4 Parameter Exploration	81
3.3 Application to Real Data: HD100453	81
3.3.1 Inclination and Position Angle of the Inner Disk	83
3.3.2 Maximum Aspect Ratio of the Inner Disk	83
3.3.3 Surface Height Profile of the Outer Disk	84
3.4 Application to Real Data: HD142527	84
3.4.1 Geometric Structure of the Inner Disk	86
3.4.2 Surface Height Profile of the Outer Disk	86
3.5 Discussion	87
3.5.1 Strengths of my Model	87
3.5.2 Limitations of my Model and its Improvement Strategies	88
3.5.3 Measuring the Cooling Time of Material in the Outer Disk	89
3.5.4 Outlook Based on the New Information Provided by my Method	91
3.6 Conclusions	93
3.7 Acknowledgment	94
Appendix 3.1: Surface Height Profile of The Outer Disk	95
Appendix 3.2: Solutions of the Shadow Boundary Curves	95
Appendix 3.3: Corner Plot of the MCMC Results	96
4 Summary	99
4.1 Summary of this thesis	99
4.2 Future works	103

Chapter 1

Introduction

1.1 Overview of Protoplanetary Disks

Protoplanetary disks are the birthplaces of planetary systems, and their evolution plays a fundamental role in shaping the architectures of emerging planets. Observations across various wavelengths have revealed diverse structures in these disks, including gaps, rings, spiral arms, and asymmetries. These features are believed to arise from processes such as planet formation, magnetic fields, and interactions between gas and dust. Millimeter-wave observations, particularly with the Atacama Large Millimeter/submillimeter Array (ALMA), have provided detailed insights into the spatial distribution of dust and gas, offering constraints on density and temperature profiles. Meanwhile, optical and infrared imaging have highlighted the scattering properties of dust grains, unveiling vertical and azimuthal structures in the disk. Together, these observations have greatly advanced our understanding of the physical and chemical conditions within protoplanetary disks, setting the stage for exploring their dynamic evolution.

1.1.1 Star Formation and Evolutionary Classification of YSOs

Stars form within dense regions of molecular clouds known as molecular cloud cores. These cores, characterized by densities of $10^3 - 10^4 \text{ cm}^{-3}$, temperatures around 10 K, and magnetic field strengths of $10 - 100 \mu\text{G}$, are gravitationally bound and provide the necessary conditions for star formation. Initially, thermal and magnetic pressures counteract gravity, but once gravitational forces dominate, collapse is triggered ([Larson 1969](#)).

This collapse initiates a dynamic, multi-phase process. In the early stages, the core contracts to form a pressure-supported structure known as the “first core”. Continued accretion causes the temperature to rise, and when it exceeds $\sim 2000 \text{ K}$ —the dissociation threshold of molecular hydrogen—a second collapse phase begins, leading to the formation of a compact, dense object

called the “second core” or protostar (Larson 1969, Tomida et al. 2010). Due to conservation of angular momentum, a rotationally supported protoplanetary disk forms around the protostar.

To observationally characterize the evolutionary stages of such young stellar objects (YSOs), the infrared spectral index α_{IR} is widely used. It is defined as:

$$\alpha_{\text{IR}} = \frac{d \log(\lambda F_{\lambda})}{d \log \lambda}, \quad (1.1)$$

where λ is the wavelength and F_{λ} is the flux density at that wavelength (Lada & Wilking 1984). This index, typically measured between 2 and 25 μm , quantifies the slope of the spectral energy distribution (SED) in the infrared and reflects the geometry, optical thickness, and thermal structure of the circumstellar material. Based on the value of α_{IR} and the shape of the SED, YSOs are classified into four main evolutionary stages: Class 0, I, II, and III, as illustrated in Figure 1.1. The following sections describe these classes in the context of the physical star formation process.

At the earliest stage, the protostar remains deeply embedded within a dense and optically thick envelope, emitting primarily in the far-infrared and submillimeter due to cold circumstellar dust. This phase corresponds to the Class 0 stage. Class 0 objects are typically not detected in near-infrared wavelengths, and their SEDs peak at long wavelengths ($T \sim 10 - 30 \text{ K}$) (Andre et al. 1993). This phase lasts roughly 10^5 years and is characterized by high mass accretion rates and the presence of powerful bipolar molecular outflows, which help to regulate angular momentum.

As the system evolves, the envelope becomes less opaque, allowing radiation from the protostar and disk to emerge at shorter wavelengths. This marks the transition to the Class I stage, during which the protostar continues to accrete material but the surrounding envelope begins to dissipate. The SED becomes flatter or even rises toward longer wavelengths, with a spectral index of $\alpha_{\text{IR}} \geq -0.3$. Class I sources often still exhibit strong molecular outflows. This evolutionary phase typically lasts for about 10^6 years.

Further evolution leads to the dispersal of the envelope, leaving behind the protostar and a well-developed protoplanetary disk. This stage is classified as Class II and corresponds to classical T Tauri stars. In this phase, infrared excess arises from the disk reprocessing stellar radiation. The SED declines in the infrared, with $-1.6 \leq \alpha_{\text{IR}} < -0.3$, and prominent emission lines in the spectrum indicate ongoing accretion from the disk. The Class II stage typically persists for a few million years and is critical for planet formation.

Eventually, the protoplanetary disk dissipates due to accretion onto the star, planet formation, photoevaporation, or stellar winds, transitioning the system into the Class III stage. These objects, known as weak-line T Tauri stars, exhibit SEDs that resemble a stellar photosphere, with little or no infrared excess. This corresponds to $\alpha_{\text{IR}} < -1.6$, indicating that most of the circumstellar material has either been accreted, incorporated into planets, or dispersed into the interstellar medium.

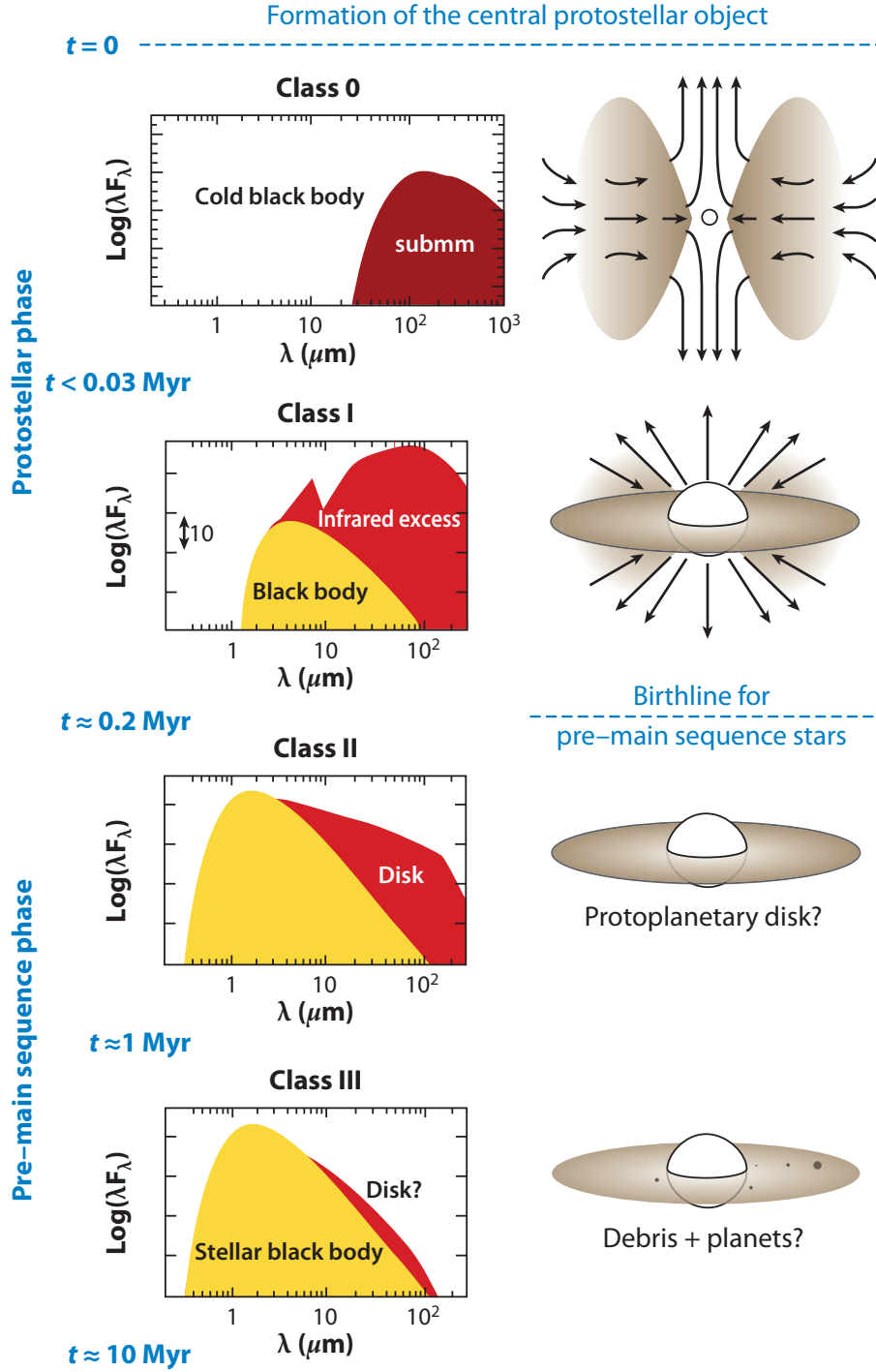


Figure 1.1: Schematic view of the common evolutionary phases of a forming star (Dauphas & Chaussidon 2011). The left panels show the SED with the star in yellow and the envelope/disk material in red, while corresponding illustrations of the disks are shown on the right. Class 0/I sources quickly evolve to pre-main sequence stars known as Class II sources, which are surrounded by protoplanetary disks. In the final stages of disk evolution, debris disks around Class III sources exhibit little excess at long wavelengths.

1.1.2 Transition Disks

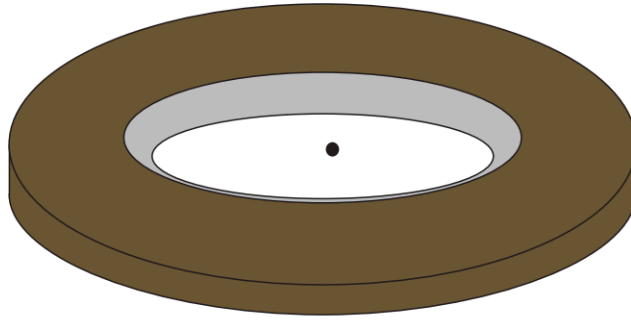
Transition disks are a class of protoplanetary disks characterized by large inner cavities formed as a result of the depletion of hot and warm dust near the central star. This dust depletion is thought to reflect ongoing disk evolution or the early stages of planet formation, and manifests clearly in the spectral energy distribution (SED). Specifically, transition disks exhibit significantly reduced emission in the near-infrared ($1\text{--}5\ \mu\text{m}$) and mid-infrared ($5\text{--}20\ \mu\text{m}$) ranges, while maintaining strong emission at longer wavelengths (Figure 1.2). These distinctive SED features serve as key indicators of cavity structures and have been widely used to identify transition disks (Espaillet et al. 2014). Prior to the advent of high-resolution millimeter interferometry, such structures were primarily inferred from unresolved photometric data using radiative transfer modeling of SEDs.

The Spitzer Space Telescope played a critical role in advancing these studies. This telescope provided mid- and far-infrared photometric data for thousands of YSOs in nearby star-forming regions. Additionally, low-resolution spectroscopy with the Infrared Spectrograph (IRS) in the $5\text{--}35\ \mu\text{m}$ wavelength range enabled more accurate identification of SED deficits. However, the spectroscopic data obtained with Spitzer IRS were limited to a small number of targets, making it difficult to conduct uniform surveys.

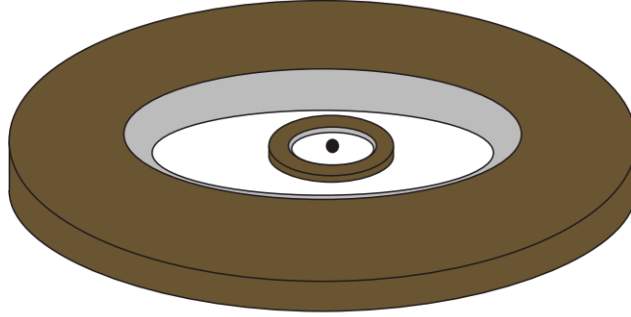
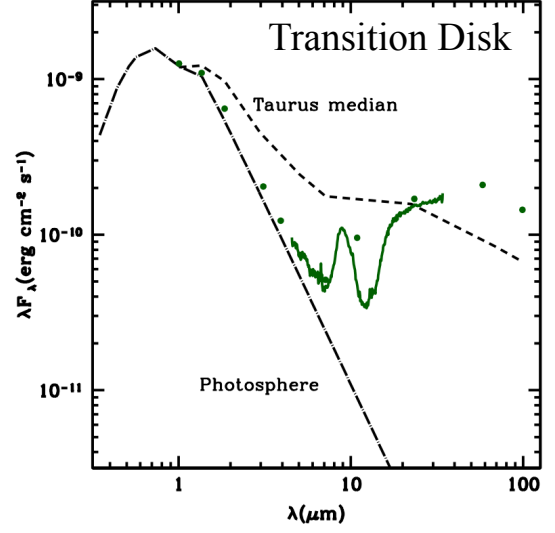
Transition disks are characterized by the absence of near-infrared excess in their SEDs, which is considered indicative of a completely cleared inner cavity. In contrast, “pre-transition disks” exhibit near-infrared excess, interpreted as the presence of an inner dust disk extending to the sublimation radius, followed by a dust-depleted gap. This classification is thought to reflect an evolutionary process in which a companion star removes inner material and gradually widens the gap over time (Espaillet et al. 2007). However, several issues challenge this interpretation:

- The near-infrared excess is primarily sensitive to the optically thick inner wall and not to the amount of dust in the inner disk.
- There is no correlation between the presence of an inner dust disk observed at millimeter wavelengths and the near-infrared excess (Francis & van der Marel 2020).
- Transition disks associated with DM Tau and TW Hya, which lack near-infrared excess, still have an inner dust ring just outside the sublimation radius, while the companion responsible for the cavity is likely located farther out (Andrews et al. 2016, Francis & van der Marel 2020).
- The near-infrared excess can vary over time, as shown by comparing Spitzer and WISE photometry for J1604.3-2130 (Zhang et al. 2014).

From this perspective, distinguishing between “pre-transition disks” and “transition disks” based solely on their SEDs becomes challenging.



Transition Disk



Pre-Transition Disk

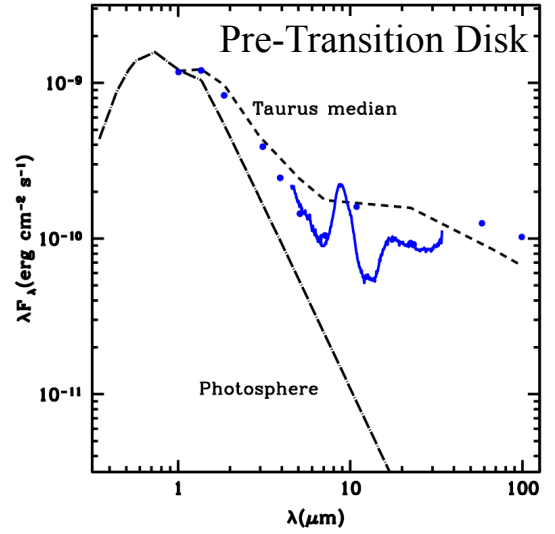


Figure 1.2: Transition disks are characterized by a large inner hole with little or no optically thick material near the star. As a result, their spectral energy distributions (SEDs) show deficits in both near-infrared (NIR; $1-5 \mu\text{m}$) and mid-infrared (MIR; $5-20 \mu\text{m}$) emission, while the emission at longer wavelengths remains comparable to that of full disks. Pre-transition disks consist of an optically thick inner disk, a relatively empty gap, and an optically thick outer disk. This structure leads to SEDs with reduced MIR emission, but NIR and longer-wavelength emission levels similar to those of full disks. This classification and interpretation follow the descriptions summarized in [Espaillet et al. \(2014\)](#).

1.1.3 Structure of the Protoplanetary Disk

Since planets form within protoplanetary disks, understanding the physical structure of these disks is essential for unveiling the overall process of planet formation. In particular, physical parameters such as temperature distribution, density structure, and the evolution of gas and dust play a critical role in determining where and how planets can form, as well as in shaping the diversity of planetary systems. The description presented here is based on (Armitage 2015).

Temperature

Several temperature models have been proposed to describe the thermal structure of passive protoplanetary disks, each reflecting different physical assumptions and levels of complexity. These models play a crucial role in understanding how the balance between stellar irradiation and radiative cooling shapes the disk's temperature distribution. In this section, we explore a selection of such models, ranging from simple analytical approximations to more sophisticated frameworks that incorporate detailed radiative transfer and dust properties.

Razor-thin Disks: The razor-thin disk model assumes a completely flat disk in the equatorial plane that absorbs all incident stellar radiation and re-emits it locally as blackbody radiation. In this simple model, the disk's temperature decreases steeply with distance from the star, following:

$$T_{\text{disk}} \propto r^{-3/4}.$$

This steep temperature gradient reflects the limited fraction of stellar radiation intercepted by the flat geometry. However, this model tends to underestimate the infrared emission of real disks, as it neglects scattering and other complexities.

Flared Disks: Flared disks feature a geometry where the height of the disk surface, h_p , increases with the radial distance r . This shape allows the disk to intercept more stellar radiation, particularly in the outer regions, leading to enhanced heating. The temperature profile for such disks is shallower, given by:

$$T_{\text{disk}} \propto r^{-1/2}. \tag{1.2}$$

This flared geometry explains the infrared excess observed in many young stellar objects, aligning with the SEDs seen in observations.

Radiative Equilibrium Disks: Radiative equilibrium disks incorporate the interaction between dust and stellar radiation, assuming a two-temperature structure. The surface layer of the disk, where stellar radiation is absorbed, is warmer than the interior. The temperature of

the dust, accounting for differences in absorption and emission efficiencies, follows:

$$T_{\text{dust}} \propto r^{-2/5}. \quad (1.3)$$

This model predicts a more gradual temperature decline compared to flared disks, which has significant implications for the outer regions' thermal state.

The Chiang–Goldreich Model: The Chiang–Goldreich model is a widely-used analytic framework that incorporates a more realistic treatment of disk physics, including dust properties and a flared geometry (Chiang & Goldreich 1997). For a disk around a star with $T_* = 4000$ K and $R_* = 2R_\odot$, the surface dust temperature is given by:

$$T_{\text{dust}} = 550 \text{ K} \left(\frac{r}{1 \text{ au}} \right)^{-2/5}. \quad (1.4)$$

This model also provides approximations for the disk interior temperature and the height of the visible photosphere. While effective in predicting the general disk structure, it underestimates midplane temperatures by neglecting accretional heating.

Vertical Density Structure

The density structure of a geometrically thin protoplanetary disk can be derived by considering the balance of vertical forces acting at a height z above the mid-plane of the disk. Such a disk orbits a central star at a cylindrical radius r . Assuming that the disk is vertically isothermal and non-self-gravitating, and that hydrostatic equilibrium is achieved in the vertical direction, the vertical pressure gradient must counteract the vertical component of the gravitational force exerted by the central star. This balance is expressed as:

$$\frac{1}{\rho} \frac{\partial \rho}{\partial z} = - \frac{\Omega_K^2}{c_s^2} z, \quad (1.5)$$

where $\rho(r, z)$ is the gas density at position (r, z) , Ω_K is the Keplerian angular velocity, and c_s is the isothermal sound speed. The Keplerian angular velocity Ω_K is defined by the gravitational force of the central star and is given by:

$$\Omega_K = \sqrt{\frac{GM_\star}{r^3}}, \quad (1.6)$$

where G is the gravitational constant and $M_\star$ is the mass of the central star. The isothermal sound speed c_s , which represents the speed of sound in an isothermal medium, is defined by the equation:

$$c_s^2 = \frac{k_B T}{\mu m_p}, \quad (1.7)$$

where k_B is the Boltzmann constant, $T(r)$ is the temperature at radius r , μ is the mean molecular weight of the gas, and m_p is the proton mass. By solving the differential equation

(1.2) for hydrostatic equilibrium, the vertical density distribution of the disk is found to follow a Gaussian profile:

$$\rho(r, z) = \frac{\Sigma(r)}{\sqrt{2\pi}H(r)} \exp\left(-\frac{z^2}{2H^2}\right), \quad (1.8)$$

where $H(r)$ is the pressure scale height and $\Sigma(r)$ is the surface density at the radius r . The Gaussian density profile indicates that most of the disk mass is concentrated near the mid-plane ($z \approx 0$). The scale height H is related to the sound speed and the Keplerian angular velocity by:

$$H = \frac{c_s}{\Omega_K}. \quad (1.9)$$

This expression shows that the scale height depends on the temperature $T(r)$ (through c_s) and the radius r (through Ω_K). In practice, the temperature $T(r)$ often decreases with increasing radius, leading to a scale height $H(r)$ that increases with radius, resulting in a flared disk structure.

Radial Structure

Radial force balance describes the equilibrium in the radial direction within a protoplanetary disk, where the competing forces acting on the gas are balanced. This analysis begins with the momentum equation for an inviscid, unmagnetized fluid. In its radial component, the equation accounts for the gravitational potential of the central star, the pressure gradient in the disk, and the centrifugal force due to rotation.

For a stationary and axisymmetric disk dominated by the gravitational potential of the star, the orbital velocity of the gas, $v_{\phi,\text{gas}}$, satisfies the following condition:

$$\frac{v_{\phi,\text{gas}}^2}{r} = \frac{GM_*}{r^2} + \frac{1}{\rho} \frac{dP}{dr}, \quad (1.10)$$

where P is the pressure, ρ is the density, G is the gravitational constant, M_* is the stellar mass, and r is the radial distance from the star. The first term on the right-hand side represents the gravitational pull of the star, while the second term corresponds to the force due to the radial pressure gradient.

In typical protoplanetary disks, the pressure decreases outward from the star, making the second term negative. As a result, the gas azimuthal velocity is slightly sub-Keplerian, meaning it is marginally slower than the Keplerian velocity of a test particle at the same radius. To quantify this deviation, the mid-plane pressure is often expressed as a power-law:

$$P = P_0 \left(\frac{r}{r_0}\right)^{-n}, \quad (1.11)$$

where P_0 is the pressure at a reference radius r_0 , and n characterizes the pressure gradient. Substituting this into the equation yields:

$$v_{\phi,\text{gas}} = v_K \left(1 - n \frac{c_s^2}{v_K^2} \right)^{\frac{1}{2}}, \quad (1.12)$$

where v_K is the Keplerian velocity and c_s is the local sound speed. The deviation from Keplerian velocity is proportional to $(H/r)^2$, where H is the vertical scale height of the disk. For geometrically thin disks, where $H/r \ll 1$, this deviation is small.

1.1.4 Evolution of the Protoplanetary Disk

Shakura–Sunyaev α prescription

A critical aspect of protoplanetary disk evolution is the mechanism responsible for angular momentum transport. While molecular viscosity ($\nu_m \sim \lambda c_s$, where λ is the mean free path and c_s is the sound speed) is the standard mechanism in terrestrial fluids, its magnitude in protoplanetary disks is far too small to account for the observed evolution timescales. For example, at 10 au, the viscous timescale due to molecular viscosity is approximately $t_\nu \sim 10^{13}$ years, orders of magnitude longer than the observed disk lifetimes.

To address this limitation, the Shakura–Sunyaev α -prescription introduces an effective, or turbulent, viscosity to describe angular momentum transport (Shakura & Sunyaev 1973). This turbulent viscosity is expressed as:

$$\nu = \alpha c_s h, \quad (1.13)$$

where h is the disk scale height, and α is a dimensionless parameter that quantifies the efficiency of turbulence in redistributing angular momentum. The parameter α is typically assumed to be less than unity but may vary depending on the disk’s temperature, density, and composition.

This α -prescription allows the viscosity to be specified in terms of local disk quantities, enabling models to compute the disk’s structure and evolution under both steady-state and time-dependent conditions. However, the approach has limitations, as α is often treated as constant for simplicity, despite its likely dependence on the physical properties of the disk gas. Additionally, turbulence, the underlying process for this effective viscosity, arises from distinct mechanisms that differ fundamentally from molecular viscosity.

Angular Momentum Transport

The evolution of protoplanetary disks is governed by angular momentum transport, which enables mass accretion. Below, the primary mechanisms are outlined along with their theoretical background and key equations.

Self-Gravity: When a disk becomes sufficiently massive, self-gravity can dominate angular momentum transport. The onset of gravitational instability is governed by the Toomre parameter Q (Toomre 1964):

$$Q = \frac{c_s \Omega}{\pi G \Sigma}, \quad (1.14)$$

where Σ is the surface density, G is the gravitational constant, and Ω is the angular velocity. If $Q < 1$, gravitational instability drives turbulence and angular momentum transport.

In this process, spiral structures form within the disk, and gravitational torques transport angular momentum outward while driving mass inward.

Magnetorotational Instability (MRI): In magnetized disks, magnetic tension introduces additional degrees of freedom, leading to instability when the angular velocity decreases with radius ($d\Omega^2/dr < 0$). This instability, known as Magnetorotational Instability (MRI), generates turbulence and facilitates efficient angular momentum transport (Balbus & Hawley 1991). The instability criterion for MRI is given by:

$$(kv_A)^2 + \frac{d\Omega^2}{d \ln r} < 0, \quad (1.15)$$

where $v_A = \sqrt{B^2/(4\pi\rho)}$ is the Alfvén speed, B is the magnetic field strength, and ρ is the density.

Disk Winds and Magnetic Accretion: Protoplanetary disk evolution has traditionally been explained by the redistribution of angular momentum within the disk itself. However, another mechanism, angular momentum loss mediated by disk winds through open magnetic field lines, offers a distinct perspective. These open field lines thread the disk and play two key roles: they exert a magnetic torque at the disk's surface, driving accretion by robbing angular momentum, and they accelerate a small fraction of the disk material outward to form a magnetically launched disk wind.

The torque per unit area exerted by the magnetic field is expressed as:

$$T_m = \frac{B_z^s B_\phi^s}{2\pi} r, \quad (1.16)$$

where B_z^s and B_ϕ^s are the vertical and azimuthal components of the magnetic field at the disk surface, respectively.

This torque induces a radial inflow velocity in the disk:

$$|v_{r,m}| = \frac{B_z^s B_\phi^s}{\pi \Sigma \Omega}, \quad (1.17)$$

where Σ is the surface density and Ω is the Keplerian angular velocity. Comparing this inflow velocity to the velocity caused by internal viscous redistribution of angular momentum, $v_{r,\text{visc}}$,

reveals that a sufficiently strong and organized magnetic field can make disk winds a more efficient mechanism for angular momentum removal than viscous processes, particularly in thin disks.

The feasibility of this mechanism depends on the strength of the vertical magnetic field threading the disk (Bai & Stone 2013). For disk winds to significantly influence disk evolution, a strong vertical magnetic field is required. However, theoretical challenges arise, such as reconciling the inward advection and outward diffusion of magnetic flux, particularly in turbulent disks.

Dust Trapping

Dust in protoplanetary disks is transported inward as a result of radial drift (Weidenschilling 1977). Radial drift is caused by the drag force acting between gas and dust particles, and its effect depends on the Stokes number, which is proportional to the particle size. However, high-resolution imaging observations of transition disks have revealed compelling evidence for the presence of dust trapping mechanisms that mitigate radial drift within protoplanetary disks (Dong et al. 2017, van der Marel et al. 2013).

Dust trapping plays a crucial role in shaping the distribution of dust. A schematic illustration of this phenomenon is shown in Figure 1.3, which depicts a planet located in the outer disk at around 20 au (marked by the vertical gray dash-dotted line) and its influence on dust dynamics. If a planet is present, it creates a gap in the disk, and at the outer edge of this gap, filtration of dust particles occurs. This process effectively traps larger particles, preventing their inward drift and leading to the accumulation of material in the outer disk.

In addition to planetary gaps, the snow line, marked by the vertical yellow dashed line in Figure 1.3, also strongly influences dust distribution. The snow line, which separates regions where water exists in solid and gaseous phases, introduces a variation in the fragmentation velocity of dust particles. Inside the snow line, where water is expected to be in its gaseous phase, the fragmentation velocity v_{frag} —defined as the threshold relative velocity above which collisions between dust particles lead to fragmentation rather than sticking—is thought to be lower. In contrast, outside the snow line, where water likely exists in solid form (as ice), v_{frag} may increase significantly.

This variation in fragmentation velocity can have a substantial impact on dust evolution in protoplanetary disks. In regions with low v_{frag} (e.g., inside the snow line), dust grains are more susceptible to fragmentation even at modest collision speeds, which suppresses their growth and leads to a population dominated by small particles. Conversely, in regions with high v_{frag} (e.g., outside the snow line), grains can withstand higher-speed collisions and continue to grow, favoring the formation of larger aggregates. Thus, the snow line introduces a key transition in the physical conditions that govern the size distribution and growth efficiency of dust particles across the disk.

Furthermore, the interplay between particle trapping and the snow line facilitates a steady

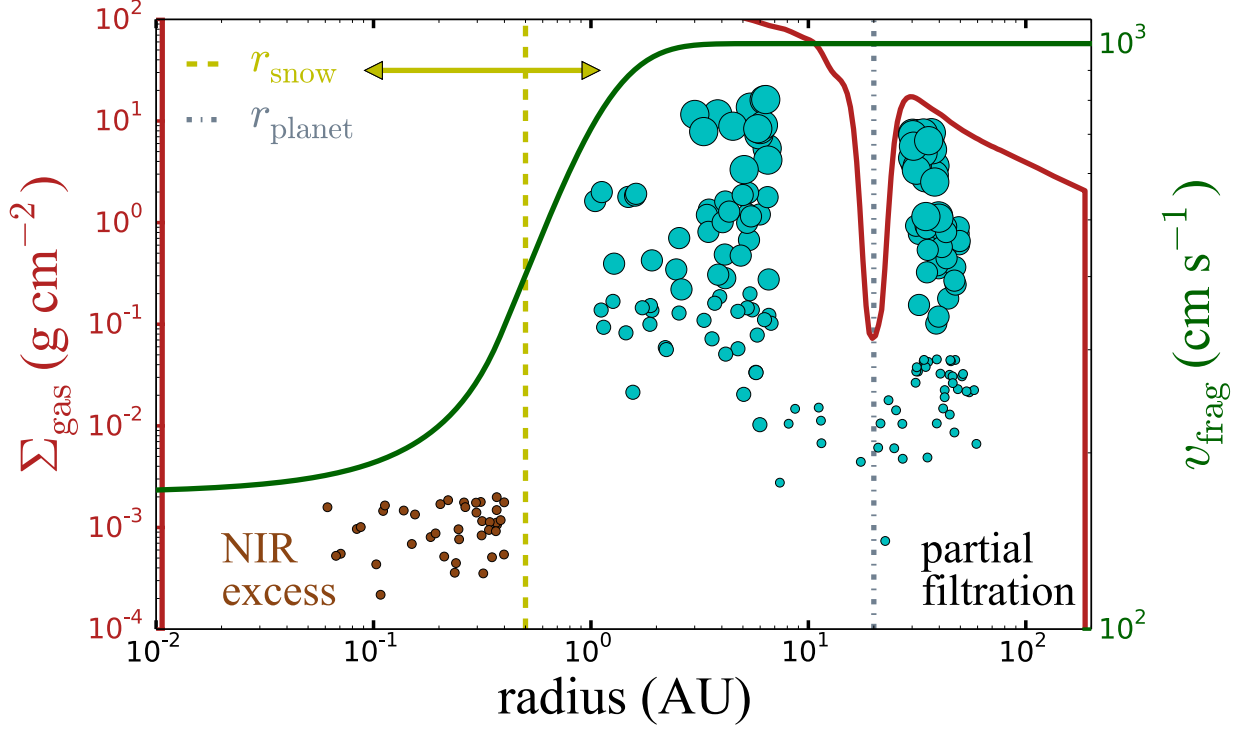


Figure 1.3: Schematic illustration of particle trapping induced by a planet located in the outer disk (around 20 au, indicated by the vertical dash-dotted line) and the influence of the snow line on dust distribution (Pinilla et al. 2016). The partial filtration of dust particles at the outer edge of the planetary gap, coupled with the variation in the fragmentation velocity of dust particles near the snow line (v_{frag} being 1 m/s inside the snow line and 10 m/s outside), facilitates a steady replenishment of small particles within the inner disk. These small grains are primarily responsible for the near-infrared (NIR) excess observed in the spectral energy distribution. The vertical dashed line marks the position of the snow line.

replenishment of small particles within the inner disk. As large particles are trapped at the outer edge of the planetary gap, smaller particles generated by fragmentation near the snow line drift inward, maintaining the population of small grains in the inner regions. The snow line itself is not static and may shift within a certain range depending on the temperature profile of the disk, as indicated by the horizontal arrow in Figure 1.3. For typical T Tauri stars, such variations in the snow line position can have profound implications for the disk’s dust evolution and subsequent planet formation processes.

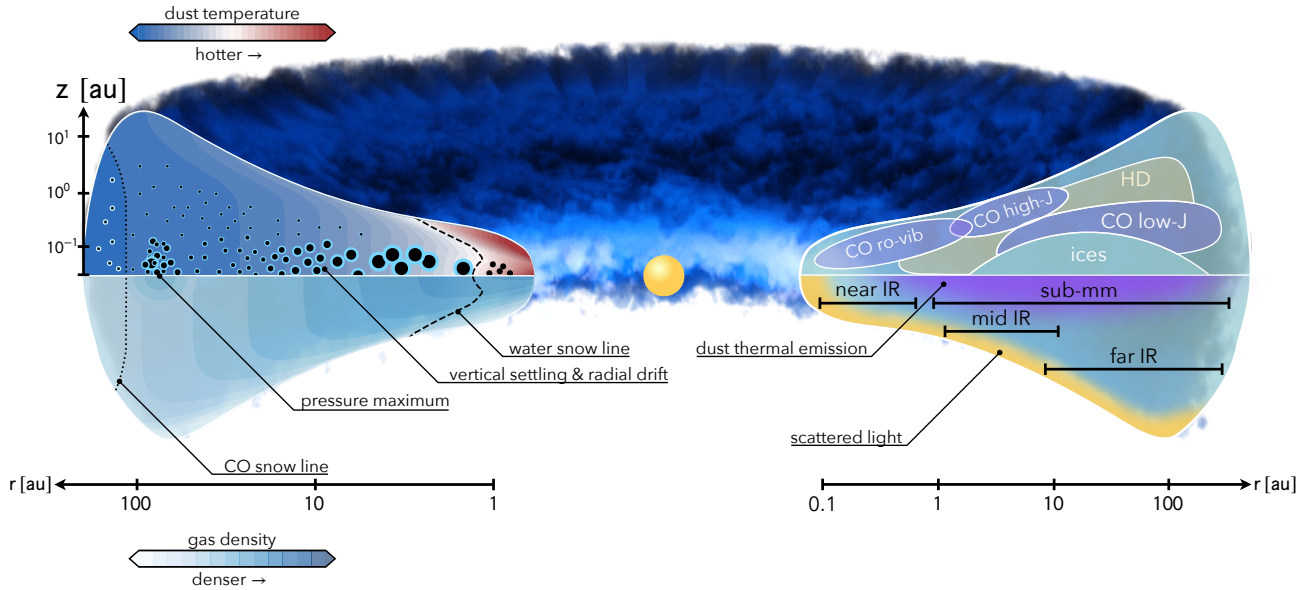


Figure 1.4: Cross-sectional illustration of a protoplanetary disk (Miotello et al. 2023). *Left* – Illustration of the dust temperature and gas density structure in protoplanetary disks. The sketched distribution of dust particles is represented by black circles, with their size variations indicated by the size of the symbols. Bare grains are present within the water snow line (dashed curve), H₂O-coated grains are outlined in blue, and CO-coated grains, located outside the CO snow line (dotted curve), are outlined in white. *Right* – The top panel shows a simplified representation of the emission regions of the main simple molecules, while the bottom panel highlights the primary regions of dust thermal emission (purple) and scattered light emission (yellow).

1.2 Observations of Protoplanetary disks

Revealing the structure of protoplanetary disks requires multi-wavelength observations to probe distinct regions and components of the disk. As shown in Figure 1.4, physical and chemical conditions vary greatly within the disk depending on position. For example, bare grains dominate the inner regions within the water snow line, while water- and CO-coated grains appear beyond the snow lines. These compositional differences affect the thermal structure and chemical environment critical for planet formation.

Different wavelengths target specific disk regions, making multi-wavelength observations essential. Millimeter and submillimeter observations, such as those with ALMA, trace the cold outer disk with high precision, while optical and infrared scattered light observations, such as those with VLT, reveal the disk’s upper layers and small dust grains. Combining these data allows us to construct a three-dimensional picture of disks, including gas and dust distribution, snow line locations, and the interaction of physical and chemical processes.

The following sections present key findings from (sub)millimeter and optical/infrared (OIR) observations, illustrating how multi-wavelength approaches have enhanced our understanding of protoplanetary disks.

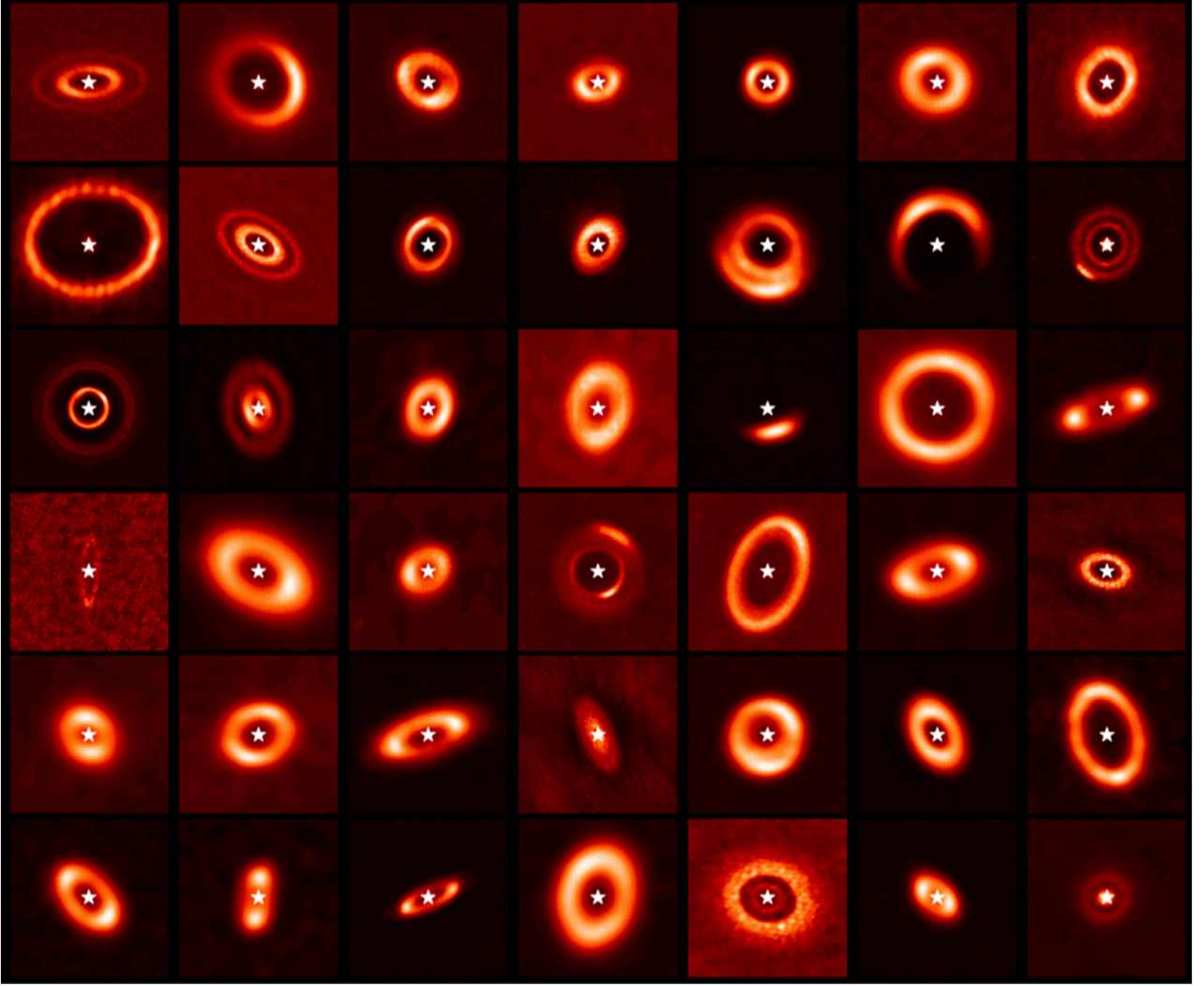


Figure 1.5: Gallery of dust continuum emission of transition disks imaged with ALMA, showing the large diversity of substructures. ([van der Marel 2023](#)).

1.2.1 (Sub)millimeter Observations with ALMA

Protoplanetary disks are fundamental structures in the study of planet formation. These disks are composed primarily of gas, with only about 1% of their mass consisting of dust and other solid particles. The physics governing their behavior and evolution is highly complex, involving a range of phenomena such as turbulence, dust dynamics, and gravitational interactions, all of which shape their structures.

Recent advances in observational technology, particularly through the use of radio interferometers such as the Atacama Large Millimeter/submillimeter Array (ALMA), have revolutionized our ability to study these disks. Interferometers work by combining signals from pairs of antennas separated by a baseline distance. The combined signals are used to measure the so-called visibility $V(u, v)$, which is the Fourier transform of the sky brightness distribution

$I(l, m)$. This relationship (van Cittert-Zernike theorem) is expressed mathematically as:

$$V(u, v) = \int \int I(l, m) e^{-2\pi i(ul+vm)} dl \, dm \quad (1.18)$$

where (u, v) are the spatial frequencies corresponding to the baseline, and (l, m) are angular coordinates on the sky. The visibilities measured by ALMA array are sampled at different spatial frequencies, effectively capturing portions of the Fourier transform of the disk’s brightness distribution. By combining these measurements, the full brightness distribution can be reconstructed with high resolution using an inverse Fourier transform. This approach allows ALMA to achieve milli-arcsec spatial resolution, revealing intricate substructures within protoplanetary disks that were previously inaccessible.

As shown in Figure 1.5, ALMA observations have revealed intricate substructures within protoplanetary disks, including concentric rings, gaps, spiral arms, and azimuthal asymmetries (van der Marel 2023, van der Marel et al. 2021). These features were beyond the reach of earlier observational capabilities and have provided unprecedented insights into the processes driving disk evolution. The structured distributions of gas and solids observed in protoplanetary disks are likely the outcomes of various physical processes, including interactions with nascent planets. Observations have revealed a diverse array of properties among different disks, particularly in the number and location of rings and gaps in the dust distribution, suggesting that the controlling mechanisms may vary between disks. This diversity indicates that stellar or disk properties could play a critical role in the processes that form these substructures.

One of the key findings from high-resolution imaging is the prevalence of concentric rings and gaps, which are commonly observed in dust continuum emissions. These features are believed to result from processes such as pressure bumps, which trap dust grains, preventing them from migrating inward (Pinilla et al. 2012). Such mechanisms may arise from a variety of sources, including the gravitational influence of forming planets, which carve gaps into the surrounding material, or instabilities in the disk itself, such as magnetohydrodynamic (MHD) effects or gravitational instability. It is still debated whether these structures are the cause or consequence of planet formation, highlighting the intricate interplay between planets and the disks that host them.

In addition to rings and gaps, protoplanetary disks often exhibit spiral arms and other asymmetries, such as crescents (Fukagawa et al. 2013, Rosotti et al. 2020). Spiral arms can be induced by the gravitational influence of a massive planet or a binary star companion, which perturbs the gas and dust, generating spiral density waves detectable with modern imaging techniques. Crescents, on the other hand, are thought to form from vortices that trap dust particles, likely caused by instabilities at gap edges or sharp transitions within the disk (Lovelace et al. 1999). These features provide valuable insights into disk dynamics and may pinpoint regions where planets are actively forming.

1.2.2 Optical and Infrared Observations

Recent advancements in optical and infrared (OIR) imaging by facilities such as the ESO Very Large Telescope (VLT), the Subaru Telescope, and the Very Large Telescope Interferometer (VLTI) have greatly enhanced our understanding of planet-forming disks and the processes driving planet formation within them. High-resolution, high-contrast imaging techniques, made possible primarily through adaptive optics (AO) on ground-based telescopes, have enabled the direct detection of protoplanets and detailed investigations of disk structures. In particular, VLTI provides milli-arcsecond resolution in the near- and mid-infrared, allowing us to probe the innermost regions of protoplanetary disks that are inaccessible to single-dish observations. These observations provide critical insights into the evolution of these complex systems.

In the OIR regimes, scattered light imaging is particularly effective in tracing small (μm -sized) dust grains, which are tightly coupled to the gas within the disk. These observations reveal complex morphological features, such as rings, spirals, arcs, and shadows, that are ubiquitous across many imaged disks as shown in Figure 1.6 (van der Marel 2023, van der Marel et al. 2021). Such features are often attributed to disk-planet interactions and provide valuable observational tests of planet formation theories.

The evolution of imaging techniques, such as Reference Differential Imaging (RDI, Grady et al. 1999), Angular Differential Imaging (ADI, Marois et al. 2006), and Polarization Differential Imaging (PDI, Kuhn et al. 2001), has played a pivotal role in enhancing the contrast and resolution of disk observations. These techniques are essential for distinguishing between stellar glare and the faint scattered light from circumstellar dust, thereby making it possible to study the fine details of disk substructures. High-contrast imaging, combined with coronagraphy and sophisticated post-processing algorithms, has significantly pushed the boundaries of what can be observed, even allowing for the study of embedded sub-stellar companions.

More recently, advanced observations with VLTI/PIONIER and VLTI/GRAVITY have enabled detailed studies of numerous Herbig stars and transition disks. These instruments can resolve structures on sub-au scales, providing unprecedented insights into the inner disk (GRAVITY Collaboration et al. 2019, Lazareff et al. 2017). Image reconstruction revealed that inner disks exhibit both axisymmetric and non-axisymmetric structures, and the location of the inner rim scales with stellar luminosity. Targeted studies with GRAVITY have also confirmed the misalignment of several inner dust disks relative to the outer disks, as traced by ALMA observations. These misalignments, detected through shadows in scattered light, have helped resolve degeneracies in inclination angle estimates (Bohn et al. 2022).

While near-infrared interferometry focuses on the inner rim of the disk, instruments like VLTI/MATISSE, which operate in the L ($3.2 - 3.9 \mu\text{m}$) to N ($8.0 - 13.0 \mu\text{m}$) bands, can trace dust emission over larger radii, up to several astronomical units. This capability is expected to provide further insights into the structure and evolution of inner dust disks.

Dust Scattering Surface

The light observed at OIR wavelength is primarily scattered light from dust, which traces the surface layers of the disk. In contrast, the radiation detected in (sub-)millimeter observations is primarily thermal emission, tracing the dust distributed near the mid-plane of the disk. Thus, OIR and submillimeter observations probe different physical regions of the disk, making it essential to integrate their results for a comprehensive understanding. In this section, we attempt a simple quantification of the height of the scattering surface observed at OIR wavelength (Muto 2011).

To calculate the scattering surface of a dust disk, we consider the radiative optical depth τ_{rad} along a path dl , defined as:

$$\frac{d\tau_{\text{rad}}}{dl} = \chi\rho \quad (1.19)$$

where χ is the opacity and ρ is the density of the medium. Integrating this expression for a vertically isothermal density profile leads to the following approximate relation for τ_{rad} :

$$\tau_{\text{rad}} \sim \chi\rho_0 r \exp\left(-\frac{z^2}{2H^2}\right), \quad (1.20)$$

where $\rho_0 = \Sigma/(\sqrt{2\pi}H)$ is the mid-plane density.

The scattering surface z_s is defined as the height where the optical depth τ_{rad} reaches a given threshold τ_s , typically set by observational constraints. Solving for z_s , we obtain:

$$z_s \sim H \sqrt{2 \log \left(\frac{\chi\rho_0 r}{\tau_s} \right)}. \quad (1.21)$$

By substituting representative values into the relevant equation, we calculate the approximate location of the scattering surface under typical disk conditions. Let the surface density be $\Sigma \sim 10 \text{ g/cm}^2$, the scale height $H \sim 10 \text{ au}$, the opacity $\chi \sim 100 \text{ cm}^2/\text{g}$, and the optical depth threshold $\tau_s \sim 1$. Substituting these values into the expression gives $z_s \sim 4H$ indicating that the scattering surface is approximately four times the pressure scale height.

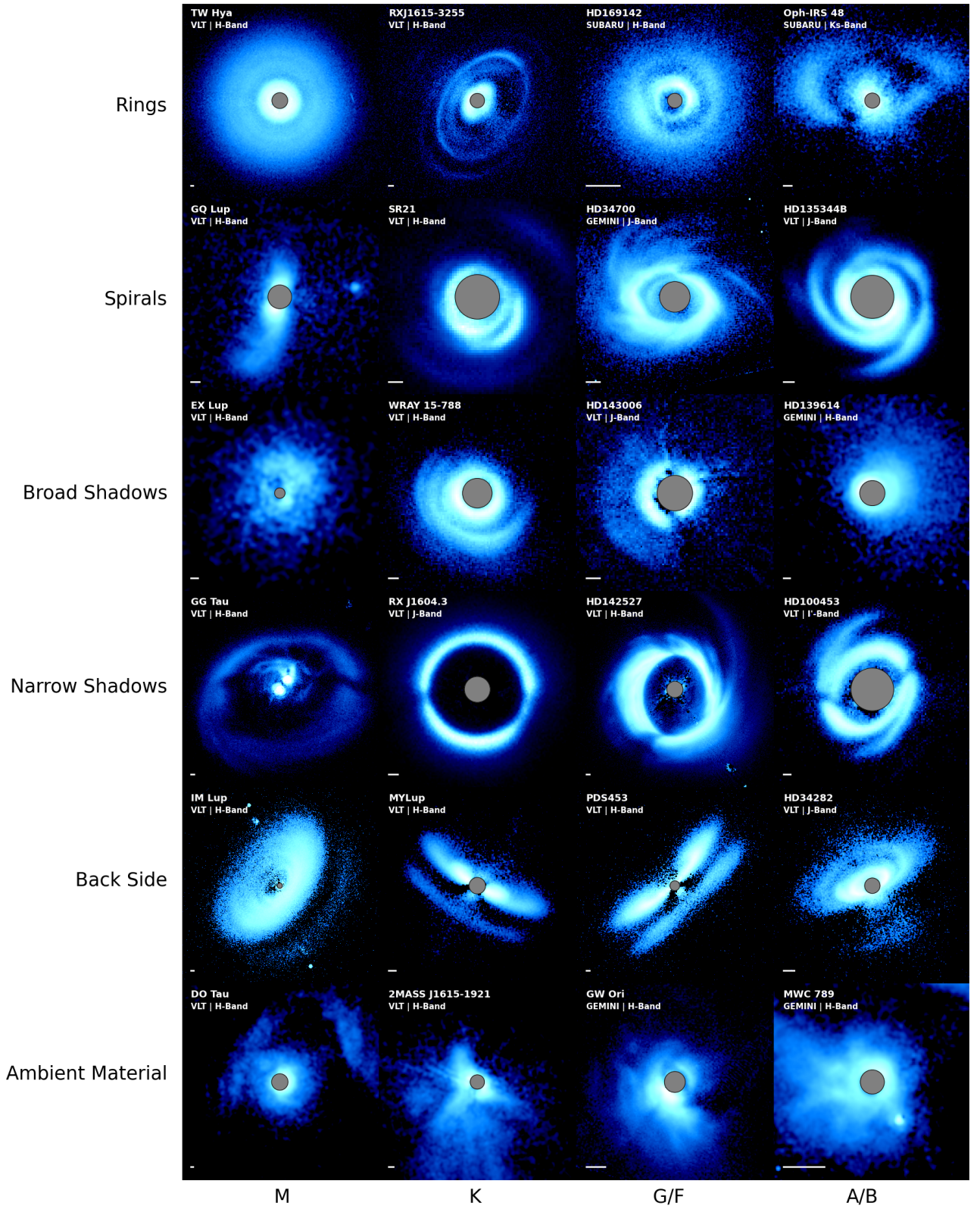


Figure 1.6: Gallery of disks observed with the VLT/SPHERE, GEMINI/GPI, and SUBARU/HiCIAO instruments, highlighting the wide range of morphological features present in planet-forming disks and their ubiquity across spectral types (Benisty et al. 2023).

1.3 Previous Studies on Inner Disks

1.3.1 Snow Line

The composition of dust within protoplanetary disks varies significantly across the water sublimation and condensation boundary, commonly referred to as the snow line. Beyond the snow line, dust primarily consists of a mixture of ice and rock. Conversely, within the snow line, water predominantly exists in its gaseous phase, and the main constituent of dust is rock.

Determining the precise location of the snow line requires a detailed understanding of the temperature structure within protoplanetary disks. Water sublimation and condensation are highly temperature-sensitive, and the snow line typically lies in regions where the temperature is approximately 170 K. In the optically thin limit of the disk, stellar radiation directly heats the mid-plane, producing a temperature distribution governed by the balance between incident energy and radiative cooling. Assuming the stellar luminosity is equivalent to the present-day solar luminosity, the resulting temperature profile follows the model proposed by [Hayashi \(1981\)](#), placing the snow line at a distance of approximately 2.7 au from the central star.

However, the temperature distribution during the planet formation stage is far more complex than this simplified model. Protoplanetary disks contain a significant amount of dust, and in their inner regions, where terrestrial planets are expected to form, the disk is typically optically thick to stellar irradiation. Consequently, stellar radiation primarily heats the disk’s surface, while the mid-plane is indirectly warmed by thermal radiation re-emitted from these heated surface layers. Because stellar radiation enters the disk surface at a steep angle, the energy received per unit area decreases in proportion to the cosine of the incident angle ([Chiang & Goldreich 1997](#)). As a result, the temperature in optically thick disks is generally lower than in optically thin disks for the same stellar parameters.

In such optically thick regions, “accretion heating” ([Lynden-Bell & Pringle 1974](#)) plays a crucial role in determining the temperature structure, complementing irradiation heating. Accretion heating occurs as gravitational potential energy released during the accretion of disk gas onto the central star is converted into heat, warming the disk. To explore this, we consider the classical model of a turbulent disk ([Shakura & Sunyaev 1973](#)). Turbulence within the disk enhances viscosity, facilitating the radial transport of angular momentum. This angular momentum transfer drives accretion, during which gravitational energy is dissipated as heat via turbulence. This specific mechanism of accretion heating is often referred to as “viscous heating”.

1.3.2 Dead Zone Inner Edge

The determination of the structure of protoplanetary disks is significantly influenced by the angular momentum transport processes of the gas. In the inner regions of the disk, where the ionization degree is sufficiently high, turbulence driven by magnetorotational instability (MRI) acts as an effective viscosity to transport angular momentum outward ([Balbus & Hawley 1991](#)).

Conversely, in the outer regions of the disk, the ionization degree decreases, creating regions where MRI becomes inactive, known as “dead zone”.

In the inner region of protoplanetary disks, the dead zone is believed to have an inner edge where the gas temperature reaches approximately 1000 K. Across the dead zone inner edge, the turbulent viscosity driven by MRI steeply decreases moving outward, creating a local maximum in the radial gas pressure profile (Flock et al. 2016). This pressure maximum acts as a trap for solid particles, promoting dust growth (Kretke & Lin 2007, Kretke et al. 2009). Moreover, the dead zone edge can also trap migrating planets by halting their inward motion (Masset et al. 2006).

According to the hydrodynamical simulations by Flock et al. (2016) and Ueda et al. (2017), the temperature and density profiles in the inner region of protoplanetary disks can be divided into four distinct regions, as shown in Figure 1.7. The temperature at the boundary between the two outermost regions (Region C and Region D) coincides with the threshold temperature required to activate MRI, suggesting that this boundary corresponds to the dead zone inner edge.

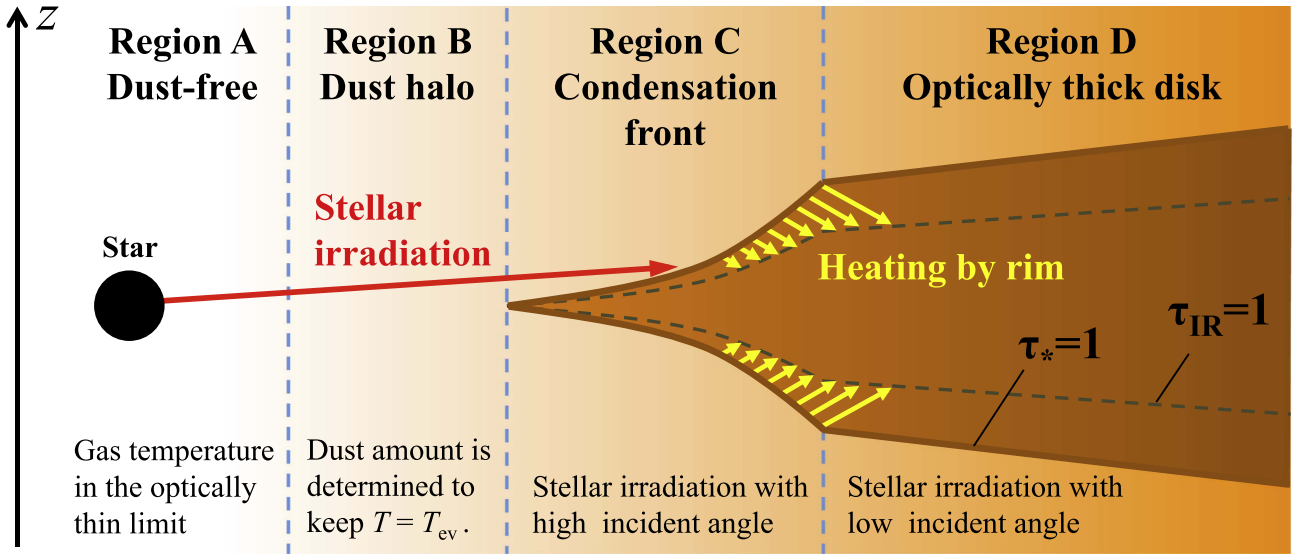


Figure 1.7: Schematic illustration of the inner structure of a passively irradiated protoplanetary disk (Ueda et al. 2017). The disk absorbs stellar radiation at the surface where the visual optical depth along the direction of starlight, τ_* , reaches unity, which is represented by the solid line. This surface acts as the primary interface for direct stellar irradiation. Deeper within the disk, the reprocessed infrared radiation originating from the $\tau_* = 1$ surface is absorbed at the deeper layer where the vertical infrared optical depth, τ_{IR} , equals unity, represented by the dashed line. The inner region of the disk can be divided into four distinct zones based on their optical and thermal characteristics. The innermost part, Region A, is a dust-free and optically thin cavity. Moving outward, Region B forms a halo where the dust is optically thin and the temperature is close to the dust evaporation temperature T_{ev} . Beyond this lies Region C, an optically thick dust condensation front that is directly irradiated by starlight at a high incident angle, resulting in a sharp temperature gradient. Further outward, Region D constitutes an optically thick area of the disk that is illuminated not only by direct starlight but also by the radiation reprocessed at the dust condensation front.

1.3.3 Misaligned Inner Disks

Misaligned inner disks are a notable features observed in some protoplanetary systems, where the inner part of the disk is misaligned with respect to the outer disk. Observations of misaligned inner disks often reveal warped or shadowed regions, where the misaligned portion casts shadows onto the outer disk. For example, the young star HD 142527 has shown evidence of shadows cast by its misaligned inner disk as shown in Figure 1.8, indicating a significant misalignment (Marino et al. 2015). High-resolution imaging, such as polarimetric observations using instruments like VLT/SPHERE, has been instrumental in detecting these features.

A misalignment between the inner and outer disks has also been confirmed from a distortion of the velocity field in the disk and can be caused by a gravitational interaction with the planet (e.g., Mayama et al. 2018; Casassus et al. 2015). Francis & van der Marel (2020) analyzed ALMA archival data of the continuum emissions from 38 transition disks and found that at least 18 transition disks in their sample harbored an inner disk in the central cavity.

In some cases, multiple moving shadows have been observed across the disk, suggesting temporal changes in the orientation of the inner disk (Pinilla et al. 2018). These moving

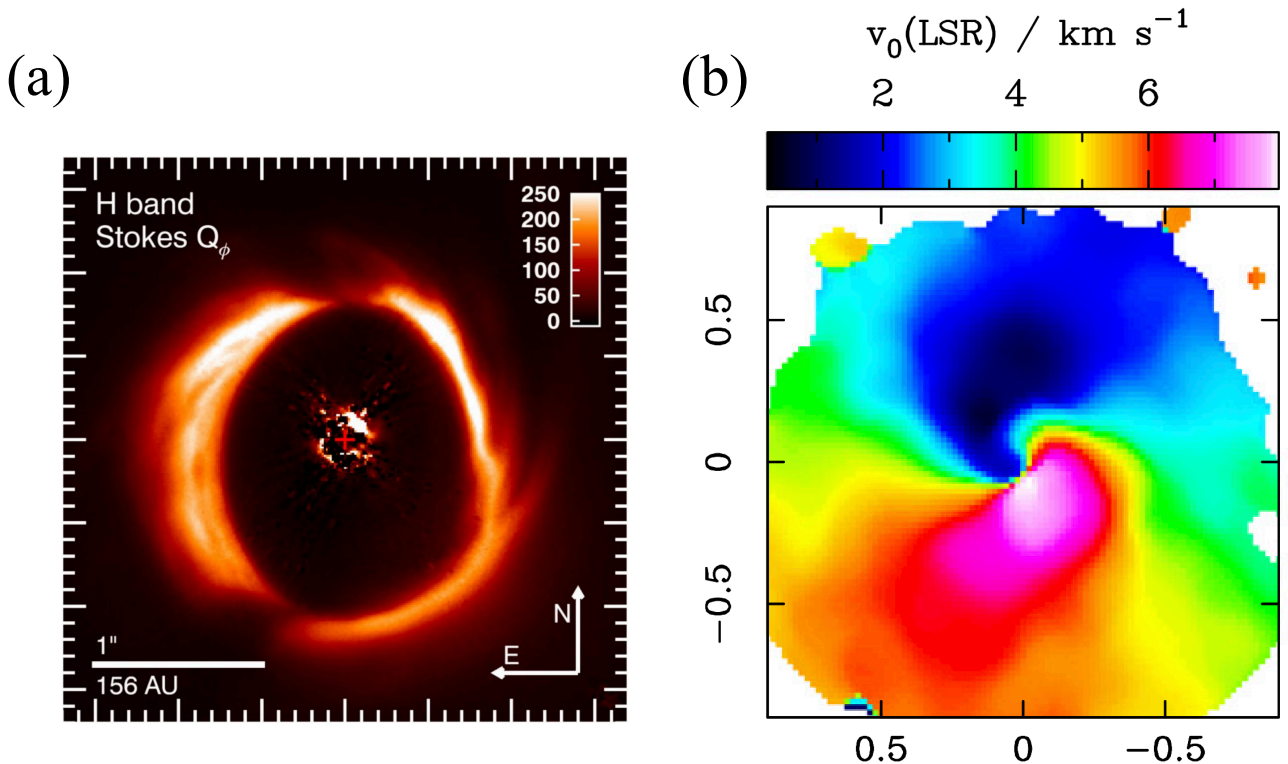


Figure 1.8: (a) Scattered light image of the HD 142527 disk taken with VLT/SPHERE in the H-band, showing prominent shadows cast by a misaligned inner disk onto the outer disk. The two dark lanes indicate regions of reduced stellar irradiation, consistent with a large misalignment between the inner and outer disks (Hunziker et al. 2021). (b) Velocity field (moment 1 map) of the CO $J = 6-5$ emission line in the same system, observed with ALMA. A pronounced deviation from a simple Keplerian pattern is evident near the disk center, suggesting the presence of a warped structure in the inner disk (Casassus et al. 2015).

shadows are particularly valuable for inferring the dynamical behavior and ongoing evolution of the misaligned inner disk.

Disk warps—variations in the orientation of a disk with respect to the stellocentric radius—are a frequently proposed phenomenon in young circumstellar disks. These warps are typically attributed to tilted inner disks that are misaligned relative to the outer regions. Numerical simulations suggest that a misaligned companion or planet can induce such warps or even fragment the disk into two or more precessing rings (Nealon et al. 2018, Nixon et al. 2013).

When the mass ratio between a companion and the primary star is high, secular resonances can lead to substantial misalignments (Owen & Lai 2017). A prominent example is the triple star system HD 100453, where a massive planet located within the cavity is thought to achieve a high inclination through the Kozai-Lidov effect (Kozai 1962, Lidov 1962). This mechanism can cause a pronounced misalignment of the inner disk (Martin et al. 2016, Nealon et al. 2020a).

Additionally, accretion events occurring after the initial collapse of the system can produce misaligned outer disks (Kuffmeier et al. 2021). Late-stage accretion, where material with misaligned angular momentum is added to the system, has also been proposed as a mechanism for outer disk misalignments (Dullemond et al. 2019).

Impact of Shadows Cast by Inner Disk on Disk Evolution

Recent studies have revealed that inner disks can be significantly misaligned with their surrounding outer disks. This misalignment causes the inner disk to cast shadows onto the outer disk, where the temperature drops locally in the shadowed regions. A notable observational feature in this context is the shadow cast within the outer disks by a tilted inner disks: HD100453 (Benisty et al. 2017), HD142527 (Marino et al. 2015), CQ Tau (Uyama et al. 2020), DoAr44 (Casassus et al. 2018), J1604 (Mayama et al. 2012). This temperature decrease creates azimuthal pressure gradients between shadowed and illuminated regions, leading to the formation of structures such as spiral arms or pressure maxima (Montesinos et al. 2016). These features enhance dust trapping and can trigger planetesimal formation in the outer disk. Understanding the impact of tilted inner disks is therefore crucial for gaining a comprehensive understanding of disk evolution and planet formation.

The mechanism by which shadows induce spiral arms was detailed by Montesinos et al. (2016). In shadowed regions, the temperature decreases, and the resulting pressure gradients cause azimuthal acceleration of the gas. This creates asymmetrical gas flows, which eventually lead to the development of spiral arms. Notably, this process can occur even in gravitationally stable disks without planets, suggesting that shadows could enable planetesimal formation across a wide range of evolutionary stages.

When inner disks are misaligned with respect to the outer disks, their shadows can exhibit dynamic motion (e.g., in the case of J1604; Pinilla et al. 2018), introducing a novel mechanism for spiral formation (Montesinos & Cuello 2018). Specifically, in the case of prograde shadow rotation, a corotation point is created where the shadow’s motion matches the orbital velocity

of the gas in the outer disk. Around this corotation point, spiral arms are triggered.

On the other hand, [Nealon et al. \(2020b\)](#) demonstrated that the precession of the inner disk causes shadows on the outer disk to rock back and forth, rather than sweeping across the entire disk (Figure 1.9). These “rocking” shadows share the same period as the inner disk precession and align with observations of strongly misaligned systems, such as the disk around J1604 ([Pinilla et al. 2018](#)). Therefore, systems with rocking shadows may have trouble establishing a corotation point, which makes spiral formation more difficult.

Furthermore, [Ziampras et al. \(2024\)](#) demonstrated through radiative hydrodynamical simulations that spiral arms excited by shadows cast by a misaligned inner disk can transport angular momentum, leading to the formation of rings and gaps. The study showed that these structures evolve on relatively short timescales under weakly turbulent conditions.

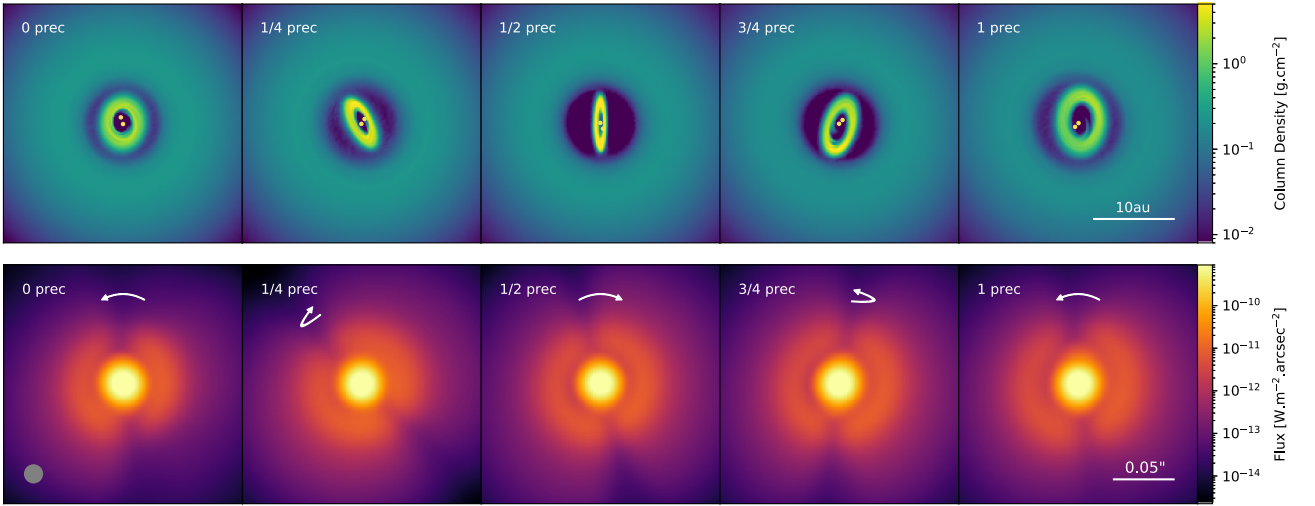


Figure 1.9: Rocking shadows in a broken circumbinary disk over one precession period of the inner disk, assuming a binary orbital plane misaligned with the outer disk by 60 degrees ([Nealon et al. 2020b](#)). The observer’s viewpoint is face-on to the outer disk. Upper row: Column density maps. Lower row: Synthetic scattered light images, with white arrows indicating the instantaneous direction of shadow motion.

1.4 Aims to This Thesis

Transition disks with depleted inner cavities often retain compact inner disks, which are increasingly found to be misaligned with respect to the outer disk. Although these inner disks are typically unresolved in direct observations, their presence and orientation can manifest through distinct features in the surrounding outer disk. These include asymmetries in temperature distributions, deviations in gas kinematics from Keplerian motion, and shadows in optical and infrared scattered light. Interpreting such observational signatures offers a promising approach to constrain the structure and orientation of otherwise unresolvable inner disks.

The primary aim of this thesis is to establish a framework for indirectly inferring the three-dimensional geometry and physical properties of inner disks by analyzing their observable influence on the outer disk. Specifically, it seeks to clarify how a misaligned inner disk affects the thermal and dynamical state of the outer disk and to determine how such effects can be used to reconstruct the structure of the inner disk itself. In this sense, the study bridges the physical connection between inner and outer disk regions through the interpretation of observational imprints.

Chapter 2 aims to observationally capture the dynamical influence of a misaligned inner disk on its surrounding environment, such as distortions in the temperature distribution and velocity field. By directly detecting and spatially resolving these perturbations, this chapter seeks to provide concrete evidence that the inclination of the inner disk can induce large-scale asymmetries in the outer disk. Such a result would be significant, as it links a small-scale structural feature—the misaligned inner disk—with global changes in disk morphology and kinematics, observable on much larger spatial scales. This provides a key step toward understanding how inner disk properties can regulate the evolution of the outer disk, and potentially influence the formation and architecture of planetary systems.

Chapter 3 focuses on clarifying the relationship between the geometric structure of the inner disk and localized cooling in the outer disk caused by shadows cast by the misaligned inner disk. To achieve this, I analyze the three-dimensional shape of the shadow projected onto the outer disk, which enables the simultaneous estimation of the geometric structures of both the inner and outer disks. Because the location and shape of the shadow depend not only on the tilt and thickness of the inner disk but also on structural parameters of the outer disk—such as its inclination and flaring angle—this approach serves as an effective means of constraining their relative configuration. Through this analysis, I quantitatively assess how a misaligned inner disk alters the thermal structure of the outer disk via shadow-induced cooling.

Finally, Chapter 4 synthesizes the results of the previous chapters to present a unified observational strategy for identifying and characterizing misaligned inner disks based on their influence on the outer disk. This chapter emphasizes that even unresolved inner disk structures can be probed through multi-wavelength observations by interpreting asymmetries in thermal, kinematic, and scattered light data.

Chapter 2

ALMA Band 6 high-resolution observations of the transition disk around SY Cha

This chapter is based on a published paper, Publications of the Astronomical Society of Japan, Volume 75, Issue 2, April 2023, Pages 424–445 ([Orihara et al. 2023](#)).

ABSTRACT

Transition disks, characterized by inner cavities within circumstellar disks, are key to understanding the processes of planet formation and disk evolution. However, the physical conditions and gas dynamics within these cavities remain poorly constrained, mainly due to limited spatial resolution in previous observations. In this study, I present the results of high-resolution (0.14 arcsec) Atacama Large Millimeter/submillimeter Array (ALMA) observations of the 225 GHz dust continuum and CO molecular emission lines from the transition disk around SY Cha. Our observations clearly resolved the disk’s inner cavity and revealed a central point source for the first time in this system. The radial dust profile can be described as a bright, narrow ring superimposed on a fainter, wider ring, and the dust continuum shows weak azimuthal asymmetry. Importantly, the gas traced by $^{12}\text{CO}(2-1)$, $^{13}\text{CO}(2-1)$, and $\text{C}^{18}\text{O}(2-1)$ is found to extend well inside the dust cavity. The $^{12}\text{CO}(2-1)$ velocity structure shows significant distortions near the inner disk, which may indicate radial gas flows or disk warping. These results contribute to a growing body of evidence that gas-rich inner disks with dynamically perturbed structures are common in transition disk systems. High-resolution studies like this one are essential to probe the gas dynamics within cavities, which is critical for understanding disk clearing mechanisms and the role of embedded planets.

2.1 Introduction

The primary aim of this chapter is to observationally capture how a misaligned inner disk influences the structure of the outer disk, which is the central theme of this thesis. As a first step, I use high-resolution observations with ALMA to spatially resolve and directly detect the inner disk, and to characterize its geometric and kinematic properties.

Inner disks with radii of a few au are particularly important in the context of protoplanetary disk evolution, as they are thought to be closely related to the formation of terrestrial planets near the central star (Francis & van der Marel 2020). In transition disks with central cavities, detecting such inner components provides essential insight into ongoing planetary formation processes.

Recent ALMA high-resolution observations have revealed an inner ring with a radius of ~ 4 au in the disk around DM Tau, which, although classified as a transition disk object (Espaillat et al. 2011; Kudo et al. 2018), exhibits features reminiscent of the pre-transition disk. This result suggests that the presence of inner disk components cannot be inferred solely from the SED, and that some transition disks may actually harbor more complex inner structures than previously assumed. Moreover, the near-infrared excess in a transition disk may not be correlated with the mass of mm-size dust in the inner disk (Francis & van der Marel 2020). Thus, high-resolution observations that reveal the actual distributions of gas and dust at a small scale are necessary to understand more about the origin of transition disks.

To detect and confirm the presence of such inner structures, it is important to consider their observable signatures. A misaligned inner disk can cast shadows onto the outer disk, leading to an asymmetric temperature distribution in the dust continuum due to uneven stellar irradiation. In addition, such misalignment may distort the velocity field of the gas, especially in CO emission lines, as the gas motion deviates from simple Keplerian rotation. These observational signatures—thermal asymmetries and kinematic distortions—serve as important clues for identifying and characterizing misaligned inner disks.

In this chapter, I report the results of high-resolution ALMA observations of the disk around SY Cha. SY Cha is a T Tauri star with the mass of $0.78 M_{\odot}$ (Manara et al. 2017). The stellar age is estimated to be ~ 3 Myr (Siess et al. 1997; Siess et al. 2000), and it exhibits a mass accretion rate of $\dot{M}_{\star} = 3.89 \times 10^{-10} M_{\odot}/\text{yr}$ (Manara et al. 2017). Other properties are summarized in table 2.1.

The SED of SY Cha (figure 2.1) does not show a significant excess in the near-infrared wavelength regime. However, for the first time, our highly sensitive and high spatial resolution observations of the circumstellar environment revealed the presence of an inner disk around SY Cha. This chapter presents observational evidence of a transition disk that harbors an inner disk.

The remainder of this chapter is structured as follows. I present the observational setup and data reduction in section 2.2, and then I report the analysis results in section 2.3. In section 2.4, I discuss the causes of the velocity distortion detected by our observations. Finally,

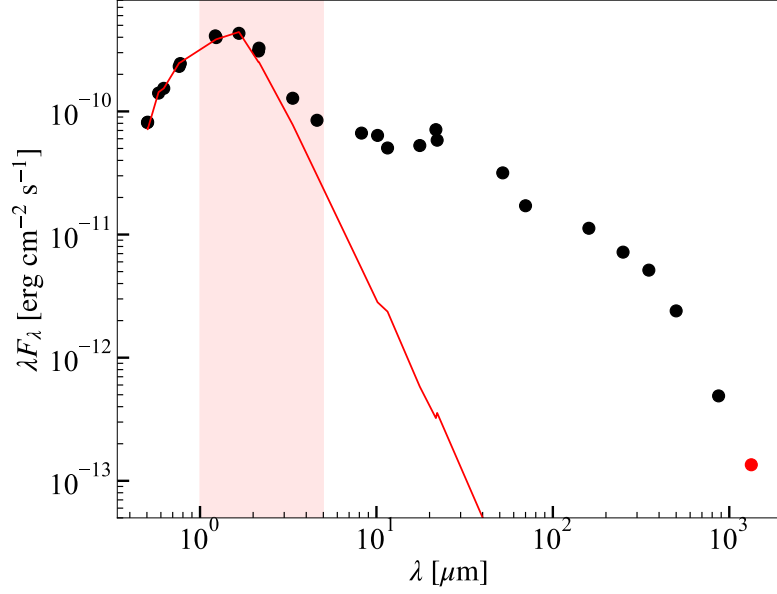


Figure 2.1: Spectral energy distribution (SED) of SY Cha (Fouqué et al. 2000; Cutri et al. 2003; Wright et al. 2010; Ishihara et al. 2010; Neugebauer et al. 1984; Winston et al. 2012; and Belloche et al. 2011). The red line indicates the photospheric emission estimated by VOSA (Bayo et al. 2008). The red circle is the flux information obtained by our observation. The red filled area indicate near-IR wavelengths (1–5 μm).

I present conclusions in section 2.5.

Table 2.1: Properties of SY Cha

Parameter	Value
Stellar Mass $M_{\star}$	$0.78 M_{\odot}$ (Manara et al. 2017)
Mass accretion rate $\dot{M}_{\star}$	$3.89 \times 10^{-10} M_{\odot}/\text{yr}$ (Manara et al. 2017)
Stellar Age	3 Myr (Siess et al. 1997; Siess et al. 2000)
Stellar Radius $R_{\star}$	$1.58 R_{\odot}$ (Gaia Collaboration et al. 2018)
Stellar Limunosity $L_{\star}$	$0.465 L_{\odot}$ (Gaia Collaboration et al. 2018)
Effective Temperature T_{eff}	3792 K (Gaia Collaboration et al. 2018)
Distance d	180.7 pc (Gaia Collaboration et al. 2021)
Spectral type	K5Ve (Frasca et al. 2015)
Proper Motion $\Delta\text{Ra.}$	-23.738 mas/yr (Gaia Collaboration et al. 2021)
Proper Motion $\Delta\text{Dec.}$	2.804 mas/yr (Gaia Collaboration et al. 2021)

Table 2.2: Observational Setup

	EC data	CC data 1	CC data 2
Obs. Date	22 June 2019	19 August 2019	25 September 2019
Configuration	C43–9	C43–7	C43–6
Number of Antenna	48	45	50
Max. Baseline [km]	16.2	3.6	2.6
Min. Baseline [m]	83.1	41.4	15.1
On-source Time [min]	72.75	15.25	15.25
Calibrators	J1107–4449 ^{1,2,3,4}		
	J1617–5848 ^{1,2,3,4}	J1107–4449 ^{1,2,3,4}	J0635–7516 ^{1,2,3,4}
	J1145–6954 ⁴	J1058–8003 ^{4,5}	J1058–8003 ⁵
	J1148–7819 ⁵	J1155–8101 ⁶	J1155–8101 ⁶
	J1155–8101 ⁶		
CASA pipeline version	42254M	42866M	42866M

¹Absolute flux calibrator, ²Atmosphere calibrator, ³Bandpass calibrator,

⁴Pointing calibrator, ⁵Complex gain calibrator, ⁶Check source

Table 2.3: Correlator Setup

	Continuum ¹	¹² CO(2–1)	¹³ CO(2–1)	C ¹⁸ O(2–1)
Central Frequency [GHz]	216.984, 232.983	230.521	220.383	219.544
Number of Channel	128, 128	3840	1920	1920
Band Width [GHz]	2, 2	0.94	0.47	0.47
Velocity width [km/s]	21.6, 20.1	0.317	0.332	0.333
Rest Frequency [GHz]	-	230.538	220.399	219.560

¹ (Lower sideband, Upper sideband)

Table 2.4: Imaging Parameters

	Continuum	Continuum taper	^{12}CO	^{13}CO	C^{18}O
Channel Width [km/s]	5330	5330	0.7	0.7	0.7
Number of Channels	1	1	25	25	25
Density Weighting (Robust)	0.5	0.5	0.5	0.5	0.5
Gaussian taper	-	1.5 M λ	1.5 M λ	0.3 M λ	0.3 M λ
Beam size	$0.040'' \times 0.022''$	$0.120'' \times 0.083''$	$0.128'' \times 0.085''$	$0.356'' \times 0.349''$	$0.356'' \times 0.348''$
Beam Position Angle	17.5°	6.3°	4.7°	-37.2°	-29.5°
RMS Noise [mJy/beam]	11.2×10^{-3}	14.9×10^{-3}	1.34	2.18	1.74
Peak Channel (V_{LSRK})[km/s]	-	-	2.0	6.2	6.2
Peak Intensity [mJy/beam]	0.347	0.962	20.9	52.8	18.4
SNR	31.0	64.6	15.6	24.2	10.6
Total Flux Density [mJy]	57.7	55.9	-	-	-
Integrated Flux Density [Jy $\cdot$ km/s]	-	-	7.9	1.4	0.3

2.2 Observations and data reduction

SY Cha was observed with ALMA in Band 6 during Cycle 6 (2018.1.00689S, PI: Muto). The observations were conducted using an extended configuration (EC) on June 22, 2019, and by compact configuration (CC) on August 19 and September 25, 2019, with the spectral windows (SPW) covering $^{12}\text{CO}(2-1)$, $^{13}\text{CO}(2-1)$, $\text{C}^{18}\text{O}(2-1)$ and continuum emission, as summarized in tables 2.2 and 2.3.

The visibility data were reduced and calibrated using the Common Astronomy Software Applications (CASA) package (McMullin et al. 2007). I used the CASA pipeline (EC data: CASA 5.40-70 42254M, CC data: CASA 5.6.1-8 42866M) to calibrate the data, as well as to flag bad data.

Two spectral windows for continuum observations with a bandwidth of 2 GHz, as well as three spectral windows for line observations, which were separated into continuum and line by the `uvcontsub` task in the CASA tools (table 2.3), were combined and averaged to obtain continuum data centered at 225 GHz. The continuum image was reconstructed by multi-scale CLEAN (Cornwell 2008) of CASA (version 6.4.4.31) with natural weighting and `scales parameters` of [0, ‘beam size’, ‘2×beam size’, ‘3×beam size’, ‘4×beam size’, ‘5×beam size’].

I performed self-calibration in phase for CC visibility using a model of the disk around SY Cha created by the `tclean` task in the CASA tools. Since the EC observation did not detect the emission with sufficient sensitivity, no self-calibration was performed for it. It is known that only correcting for proper motion is sometimes inadequate and the alignment of ALMA data taken with different execution blocks is required (Hashimoto et al. 2021). To correct for misalignment of the phase center between observations, the phase centers of all observations were set at the position where the root mean square (RMS) of the imaginary part of the visibility was minimized by the `phaseshift` and `fixplanets` tasks in the CASA tools, taking advantage of the fact that the disk is nearly axisymmetric (Kanagawa et al. 2021). The real and imaginary parts of the visibility correspond to the symmetric and asymmetric components, respectively, of the image with respect to 180° rotation. Here, because it is not necessarily true that the asymmetry would behave similarly on different spatial scales, the above process was limited to a spatial frequency range of $300 \sim 1000\text{k}\lambda$, which was a common sample for each observation. The final round of self-calibration in phase was then performed with `combine` “spw, scan” and `solint` “inf” for concatenated visibility. This corrects for the slight difference of the phase center between observations. Finally, I obtained a dust continuum image with a beam size of $0.040'' \times 0.022''$ and an RMS noise of $11.2 \mu\text{Jy/beam}$. However, this image is not sensitive to weakly extended emissions, so I also prepared the tapered image with a beam size of $0.120'' \times 0.083''$ and an RMS noise of $14.9 \mu\text{Jy/beam}$.

Continuum components were subtracted from all line data in the visibility domain with the task `uvcontsub` in the CASA tools. Before imaging the line emission, the solution tables obtained from the self-calibration of the continuum data were applied to the line data. Channel maps with a velocity width of 0.7 km/s were created by Hogbom CLEAN in a velocity range of

$V_{\text{LSRK}} = -4.3\text{--}12.5$ km/s. Subsequently, I set the density weight and scale parameters similar to the continuum imaging. Because $^{12}\text{CO}(2\text{--}1)$ emission was detected in the EC data, I performed imaging using the concatenated visibility. The beam size and RMS noise of each channel map are presented in table 2.4. To enhance the SNR, I applied a Gaussian uvtaper of $1.5\text{ M}\lambda$ on the ^{12}CO , and $0.3\text{ M}\lambda$ on the ^{13}CO and C^{18}O . The beam size of the highest-resolution image is $0.128'' \times 0.085''$.

I created the moment maps from each channel map using the `bettermoments` package (Teague & Foreman-Mackey 2018). Here, the channel maps were applied to the Keplerian mask generated with the `keplerian_mask` package (Teague 2020) before creating the moment map. This package can calculate masks based on the expected emitting surface of a Keplerian disk. The calculation assumed the stellar mass $M_{\star} = 0.78\text{ M}_{\odot}$ (Manara et al. 2017), distance to the star $d = 180.7$ pc (Gaia Collaboration et al. 2021), and applied the position angle (PA) and inclination obtained from our continuum observations (see section 2.3.1). In addition, I assumed the height of the emitting surface relative to the radius to be 0.13 from the estimation of scale height (see section 2.3.3). The mask was convolved with a 2D Gaussian that has the same size as the synthesized beam of each channel map. When creating moment 1 and 2 maps, the clipping levels were set to 3σ (1σ is the RMS of the emission free region of each channel map).

2.3 Data analysis and results

2.3.1 Synthesized images

225 GHz dust continuum

The dust continuum image obtained by MS-CLEAN is shown in figure 2.2. The continuum emission has a total flux density of 55.9 mJy that was measured above 3σ in the emission. The emission consists of a ring and a central source of 0.4 mJy. The 2D Gaussian fitting of the point source on the image plane using `emcee` (Foreman-Mackey et al. 2013) shows that this size is $42 \pm 0.96\text{ mas} \times 25 \pm 0.37\text{ mas}$ and is comparable in size to the beam, thus the central source is unresolved. This size can be regarded as the upper limit on the diameter of the inner dust disk.

To characterize the ring structure, the center and tilt of the disk were determined. I consider that the ring center is not necessarily located at the position of the central point source. Even when the physical center of the dust ring is located at the stellar position (and also the central component), the emergent emission from the ring can show asymmetry due to near-far asymmetry along the minor axis. Therefore, I determined the center of the dust ring using the following methods. First, I removed the emission corresponding to the unresolved inner disk at the center of the image by subtracting 0.4 mJy from the real part of all of the observed visibility. Secondly, I searched for the center of the ring structure. This was performed in the visibility domain by shifting the phase center and minimizing the imaginary part of the concatenated

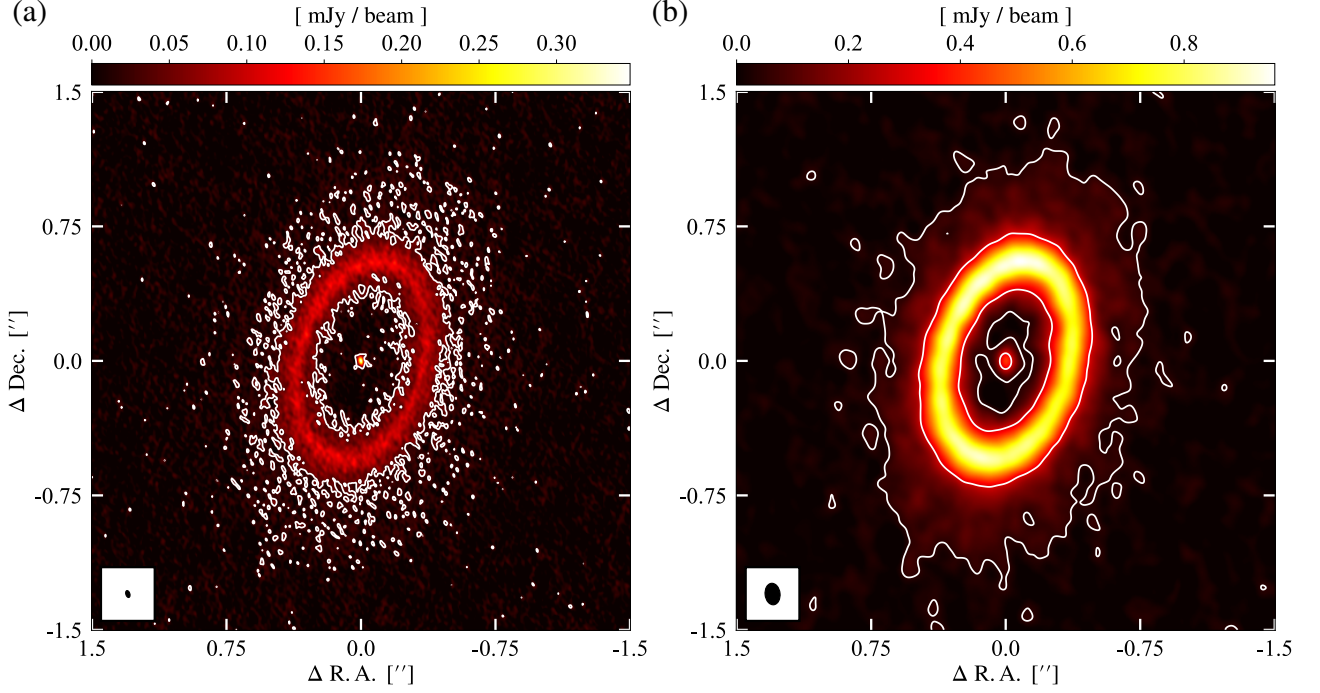


Figure 2.2: 225 GHz dust continuum images around SY Cha. (a) The synthesized beam size ($0.040'' \times 0.022''$) is shown at the bottom left. The white contour denotes the 3σ , where the 1σ level is the RMS noise ($= 11.2 \mu\text{Jy beam}^{-1}$). (b) The image was made by adding uvtaper to the `tclean` parameter. The synthesized beam size ($0.120'' \times 0.083''$) is shown at the bottom left. The white contour denotes the 3σ , 20σ , where the 1σ level is the RMS noise ($= 14.9 \mu\text{Jy beam}^{-1}$).

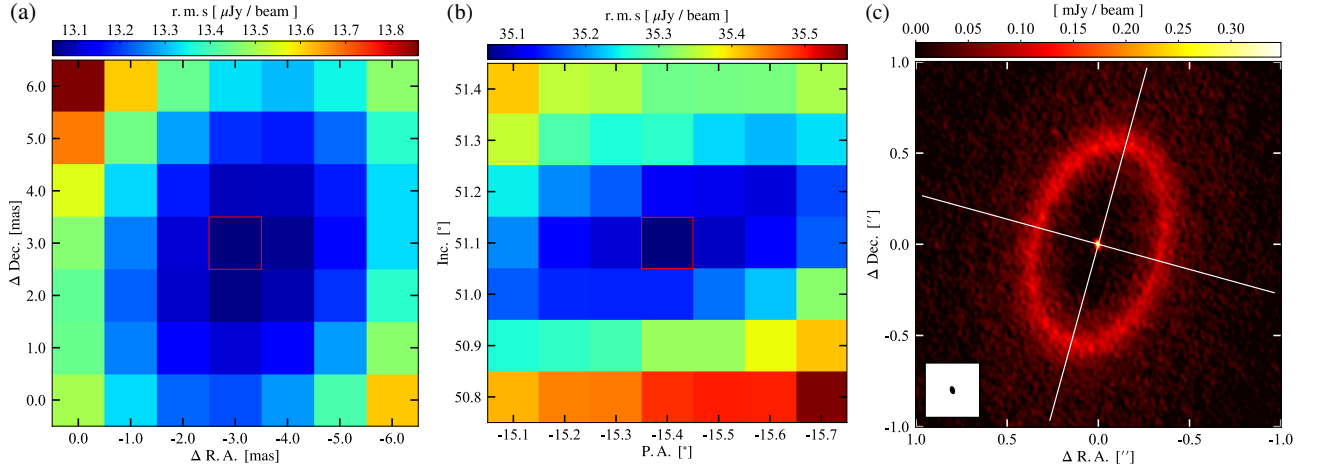


Figure 2.3: (a) RMS of the images synthesized with only the imaginary part of the visibility by shifting in steps of 1 mas in R.A. and Dec directions. The axes show a shift from the phase center of the point source. The RMS is calculated in the region with a radius of 2 arcsec. The phase center shifted by -3 mas in the RA direction and 3 mas in the Dec direction minimizes the RMS of the image made from the visibility with real part = 0 (red box). (b) RMS of difference between the image and its 90° -rotation. The images were synthesized with the deprojected visibility by changing 0.1 deg in PA and inclination. The deprojection with $\text{PA} = -15.4^\circ$ and $\text{Inc} = 51.1^\circ$ minimizes the RMS (red box). (c) The white solid line denotes the major and minor axes of the disk.

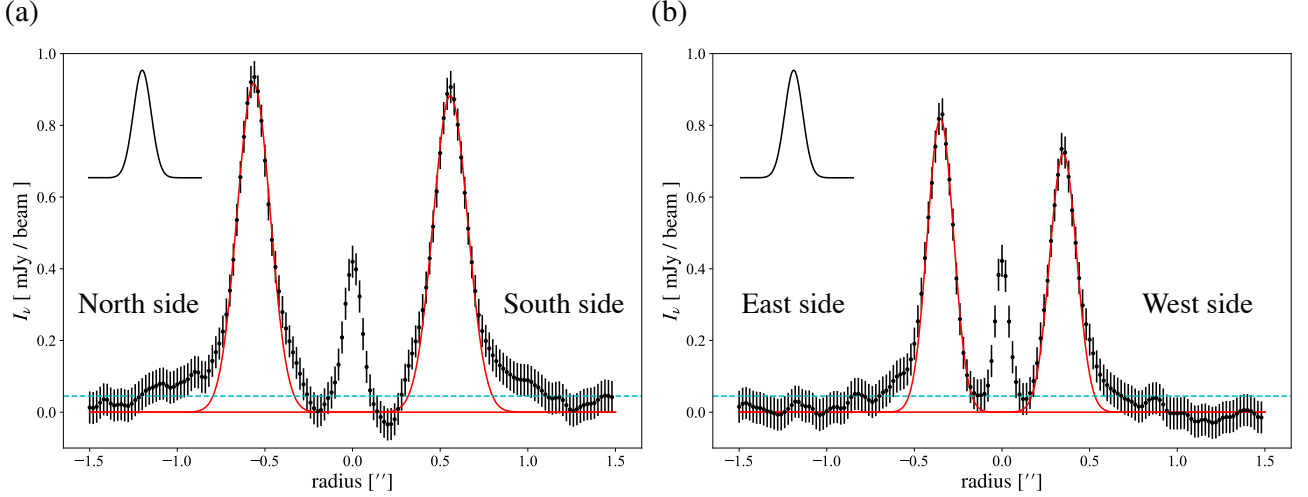


Figure 2.4: Radial profiles of the intensity along the (a) major and (b) minor axes (figure 2.2b). The red line shows the best-fit curve in the Gaussian fitting at each peak. The error bar indicates the 3σ , where the 1σ level is the RMS noise ($= 14.9 \mu\text{Jy beam}^{-1}$). The black curve in the top left indicates the profile of the synthesized beam. The cyan dashed horizontal line is 3σ .

visibility; in this process, all visibilities with any spatial frequencies were used. Figure 2.3a shows the RMS level in each map synthesized with only the imaginary part as a function of the shift of phase center. The RMS values of the images synthesized with only the imaginary part of the visibility were measured in the central region with a radius of 2 arcsec. The minimum value of RMS was obtained for the phase center shift $\Delta\text{R.A.} = -3 \text{ mas}$ and $\Delta\text{Dec.} = 3 \text{ mas}$. Finally, I searched for the tilt of the disk. This was performed by minimizing the RMS of the difference between the image and its 90° rotation. If the image is completely deprojected to face-on, the difference should be minimized.

Figure 2.3b shows the RMS level in each map of difference between the image and its 90° -rotation as a function of PA and Inclination. The result presented in this chapter was obtained when the image was rotated 90° to the east, but the same result was obtained when the image was rotated to the west. The minimum value of RMS was obtained for the deprojection with $\text{PA} = -15.4^\circ$ and $\text{Inc} = 51.1^\circ$.

The surface brightness profiles along the major and minor axes are presented in figure 2.4. I fit these profiles with three Gaussian components. Two of them correspond to the ring, and the remaining corresponds to the central emission. The results of the Gaussian fitting along the major and minor axes are shown in tables 2.5 and 2.6, respectively. The FWHM of the outer ring is about twice the beam of the dust continuum image. I therefore consider that the width of the outer ring is marginally resolved. I found that the radial profiles at the outer radii ($r \gtrsim 0.5 \text{ arcsec}$) do not follow the Gaussian distribution. According to the profile along the major axis, there is a weak emission ($> 3\sigma$) that extends to $\sim 1.2 \text{ arcsec}$ from the central star. More detailed analyses of the brightness distribution of dust continuum emissions in the visibility domain are presented in section 2.3.2.

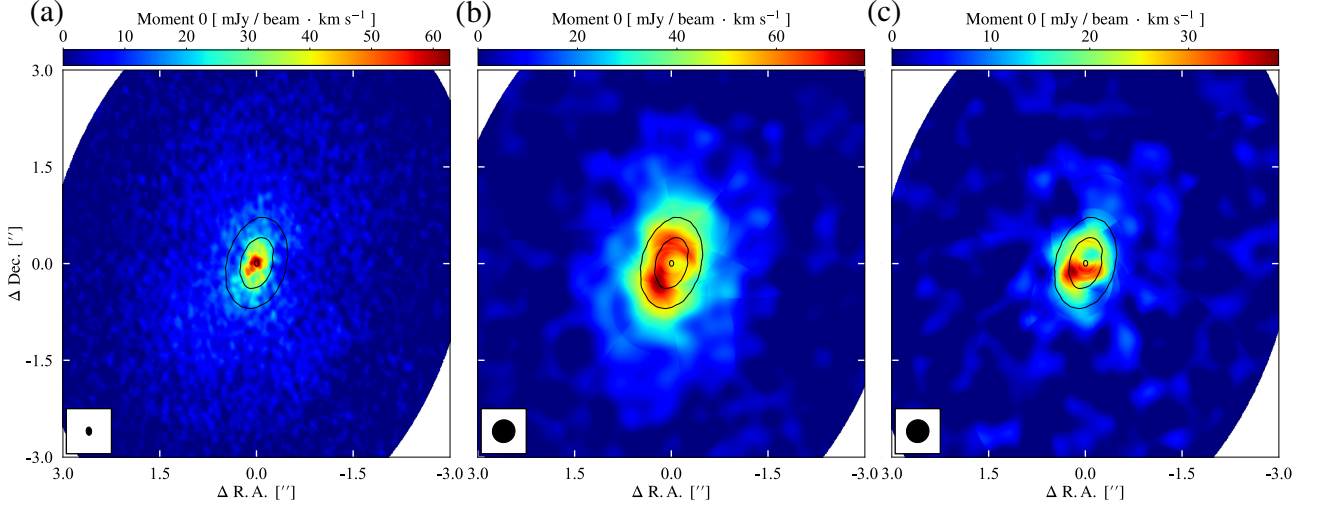


Figure 2.5: Moment 0 maps of (a) $^{12}\text{CO}(2-1)$, (b) $^{13}\text{CO}(2-1)$, (c) $\text{C}^{18}\text{O}(2-1)$. The black ellipse at the bottom left of each panel represents the beam size. The contours in these maps indicate the 20σ level of the dust continuum, where $1\sigma = 14.9 \mu\text{Jy}/\text{beam}$.

Table 2.5: Results of Gaussian fitting at the major axis

Parameter	North	South
$r_{\text{peak}}[\text{arcsec, au}]$	$-0.566, -102.3$	$0.559, 101.0$
FWHM[arcsec, au]	$0.216, 39.0$	$0.225, 40.7$
$I_{\nu}[\text{mJy}/\text{beam}]$	0.922	0.889

Table 2.6: Results of Gaussian fitting at the minor axis

Parameter	East	West
$r_{\text{peak}}[\text{arcsec, au}]$	$-0.351, -63.4$	$0.349, 63.1$
FWHM[arcsec, au]	$0.178, 32.2$	$0.183, 33.1$
$I_{\nu}[\text{mJy}/\text{beam}]$	0.815	0.717

In the radial profile along the minor axis, the peak on the east side is $\sim 10\%$ higher than that on the west side. This difference corresponds to 6.6σ ($1\sigma = 14.9 \mu\text{Jy}/\text{beam}$); thus, it is statistically significant. The ring emission may be asymmetric along the minor axis, probably because the eastern side is the far side of the disk. The inner wall of the ring, which has a slightly higher temperature than the rest of the ring due to stellar irradiation, is observed in the eastern side (see section 2.4 for further discussion).

CO line

The moment 0 (integrated intensity) maps of the CO line emission are shown in figure 2.5. Each panel corresponds to ^{12}CO data, ^{13}CO data, and C^{18}O data. I summarized the integrated

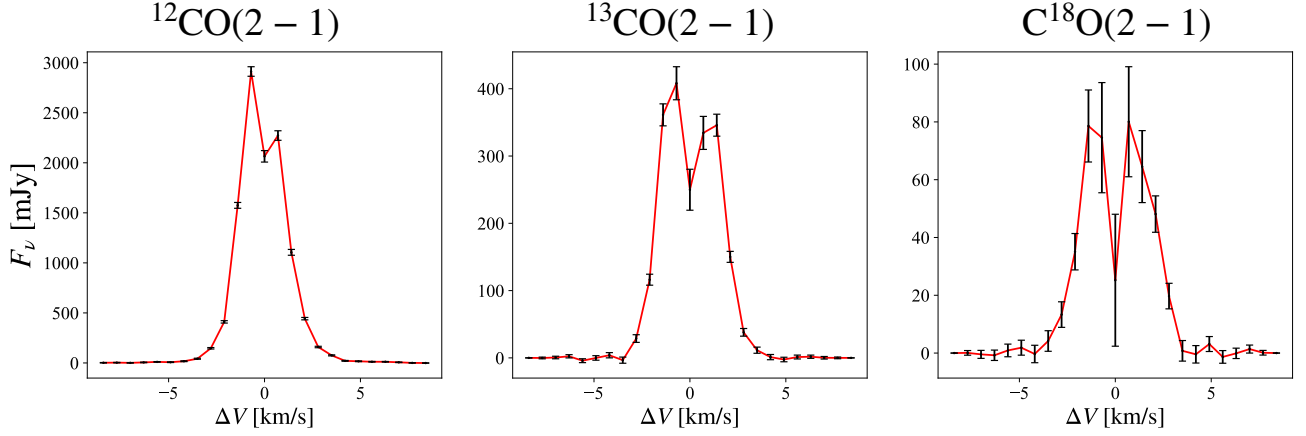


Figure 2.6: $^{12}\text{CO}(2-1)$, $^{13}\text{CO}(2-1)$, and $\text{C}^{18}\text{O}(2-1)$ emission line spectra integrated within the Keplerian mask. The error bars were obtained by error propagation from the image RMS.

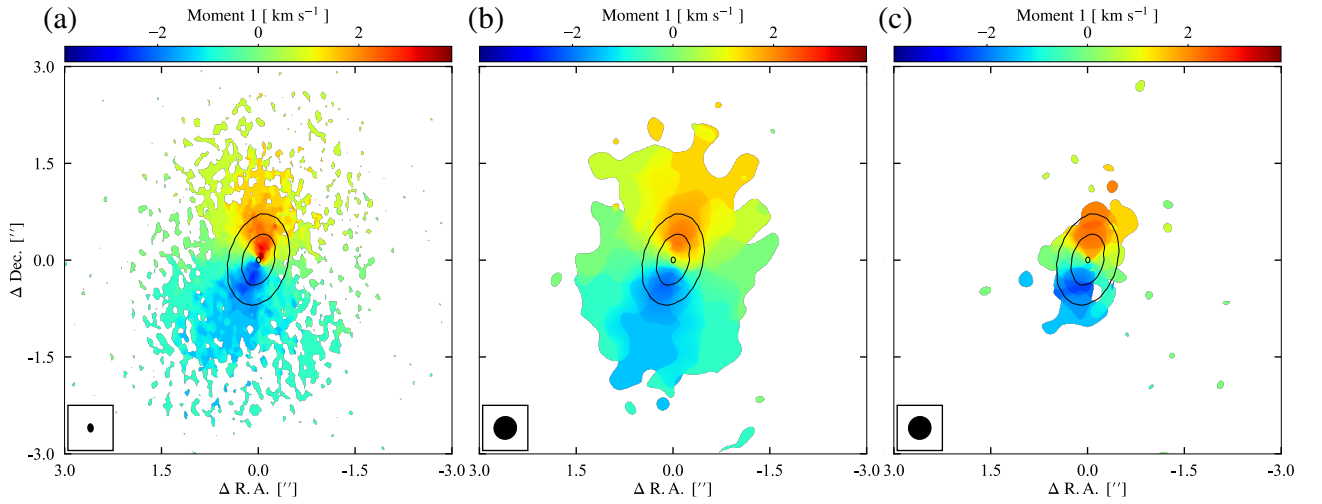


Figure 2.7: Moment1 maps of (a) $^{12}\text{CO}(2-1)$, (b) $^{13}\text{CO}(2-1)$, (c) $\text{C}^{18}\text{O}(2-1)$. The black ellipse at the bottom left of each panel represents the beam size. These color bars show the velocities based on systemic velocity $V_{\text{LSRK}} = 4.1 \text{ km/s}$. The contours in these maps indicate the 20σ level of the dust continuum, where $1\sigma = 14.9 \mu\text{Jy/beam}$. When creating the moment 1 maps, the clipping levels were set in 3σ .

flux of each moment 0 map in table 2.4. The ^{12}CO line shows that the line emission comes from the elliptical region whose major axis well matches with that of the dust ring. I observe the ^{12}CO emission inside the dust ring, which peaks at the location where the dust emission at the center of the ring is observed. The moment 0 map of the ^{13}CO line emission also shows an extended distribution along the major axis of the dust ring. Contrary to the ^{12}CO emission, the ^{13}CO emission is concentrated in the inner edge of the dust ring, and also shows an east-west asymmetry, as in the case of the dust continuum. This suggest that the disk is optically thinner for the ^{13}CO emission, tracing both column density and temperature, and the peak of the image seems to delineate a high gas density near the dust ring. The distribution of C^{18}O emission resembles that of ^{13}CO , but the signal is less significant; the intensity of the C^{18}O emission in

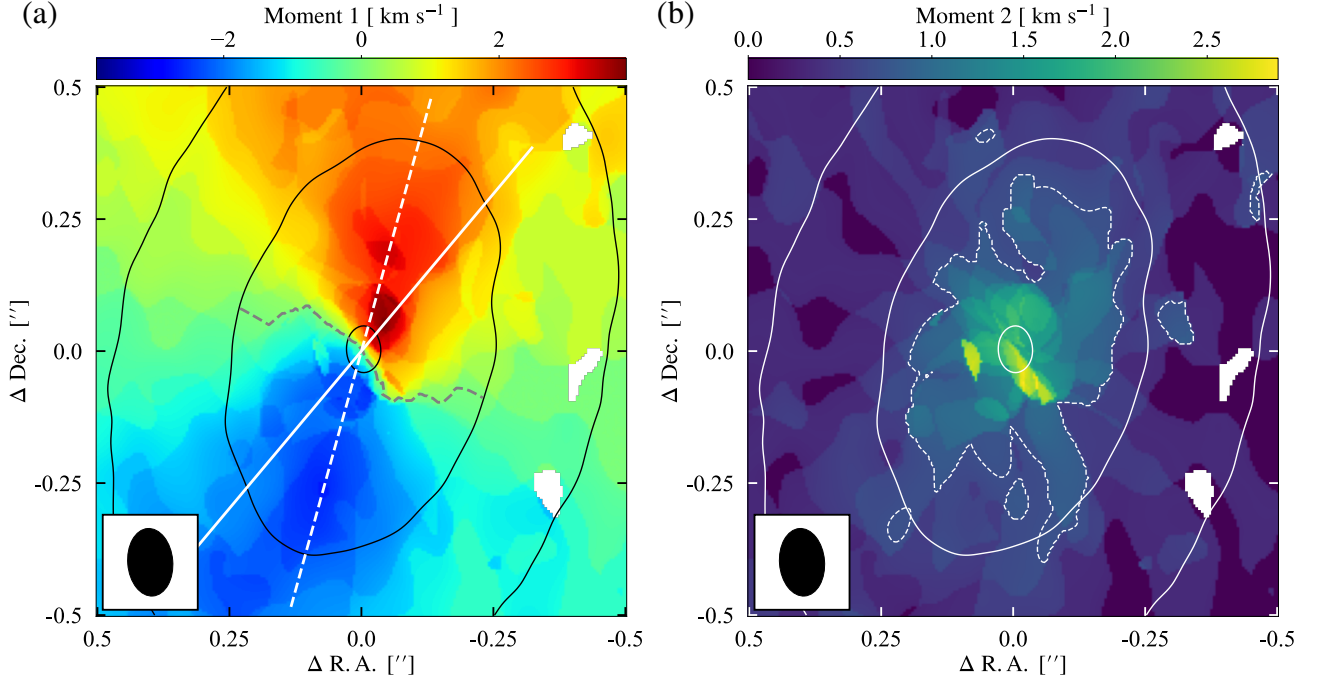


Figure 2.8: (a) Zoom-up moment 1 map made from the $^{12}\text{CO}(2-1)$ emission. The black ellipse at the bottom left represents the beam size. The gray dashed contour indicates the systemic velocity. The dashed and solid lines denote the major axis of the outer ring and an inner disk (see the discussion in section 2.4.2), respectively. (b) Moment 2 map of $^{12}\text{CO}(2-1)$. When creating the moment 2 map, the clipping levels were set at 3σ . The synthesized beam size is shown at the bottom left. The dashed contours indicate the velocity dispersion of 0.7 km/s . The solid contours in these maps indicate the 20σ level of the dust continuum, where $1\sigma = 14.9 \mu\text{Jy/beam}$.

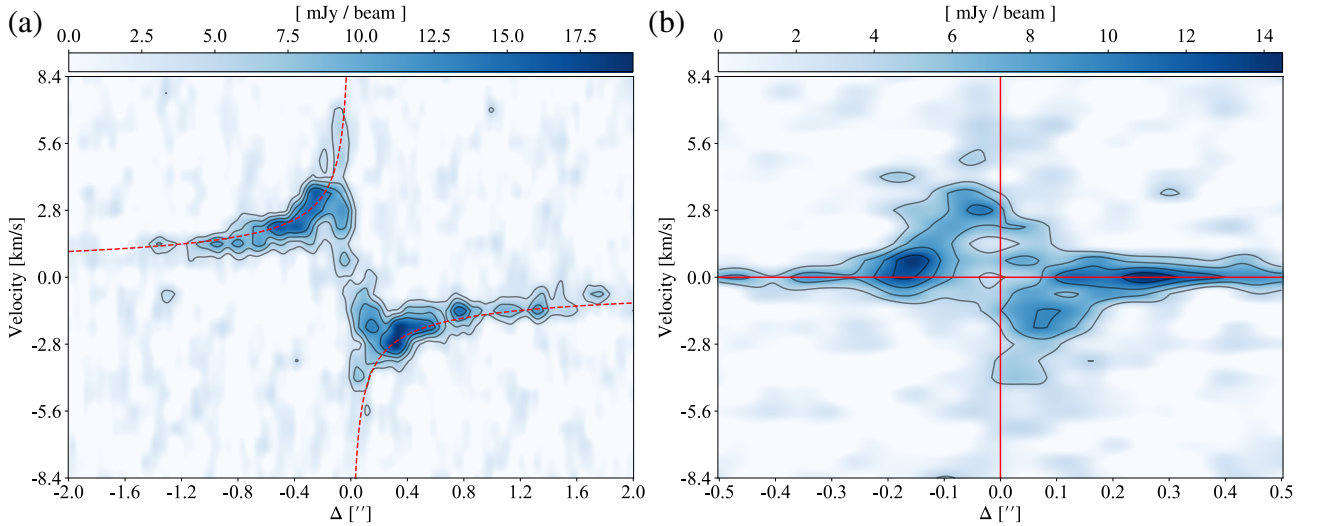


Figure 2.9: PV diagram made from the $^{12}\text{CO}(2-1)$ cube along the (a) major and (b) minor axes of the outer ring. Contours are shown at $[3,5,7,9]\sigma$, where the 1σ level is 1.34 mJy/beam . (a): The red curves show the Keplerian velocity curve for the central star of $0.78 M_{\odot}$, and an outer disk inclination of 51.1° . (b): The solid red lines denote the center position, and the systemic velocity of 0. There exists a velocity gradient in the vicinity of the center.

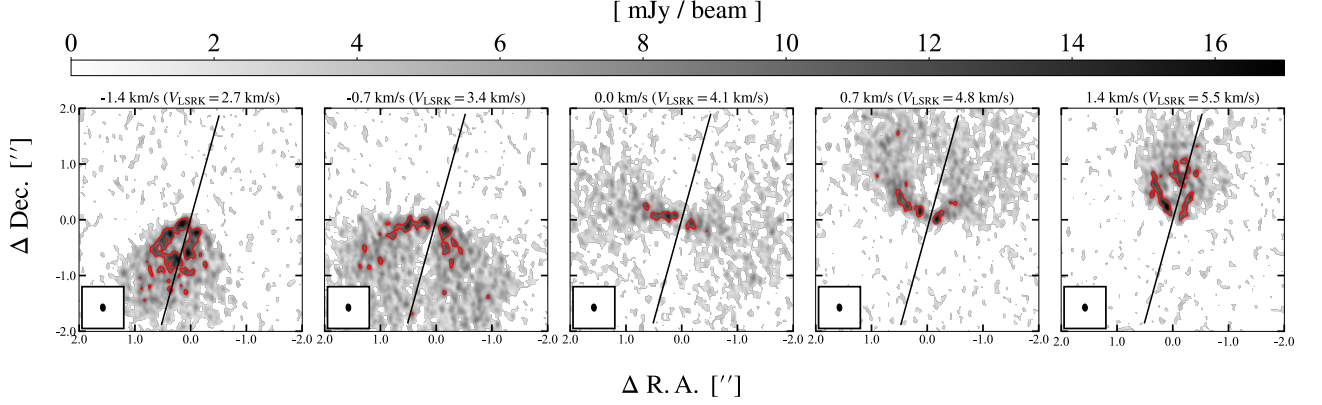


Figure 2.10: The butterfly patterns on panels of $\Delta V = -1.4 - 1.4 \text{ km/s}$. The red contours indicate the 10σ level of the $^{12}\text{CO}(2-1)$, where $1\sigma = 1.34 \text{ mJy/beam}$. The black lines denote the major axis of the disk.

channel maps has at most 10.6σ with 0.35 arcsec beam, which is 2.3 times lower than that of the ^{13}CO emission.

As shown in figure 2.6, all line emissions indicate the spectra integrated within the Keplerian mask. in which the blue channels are brighter than the red channels. In the case of absorption by the foreground cloud, it is relatively optically thick for the ^{12}CO line, resulting in a relatively high extinction of the ^{12}CO line spectra. However, since all the emission lines show similar spectra, this is most likely a result of the gas distribution in the disk rather than absorption by the foreground cloud.

Figure 2.7 shows the moment 1 (intensity-weighted velocity) map of CO lines. The velocity clearly shows the rotation pattern, and the direction of the large velocity gradient coincides with that of the major axis of the dust ring. The velocity structure revealed in the moment 1 maps of both ^{13}CO and C^{18}O agree with that of ^{12}CO , which is consistent with rotation in a disk.

However, I see a peculiar velocity pattern in the vicinity of the central star. Figure 2.8a is the moment 1 map of ^{12}CO emission near the central star. Although the line-of-sight velocity should be zero along the minor axis of a coplanar disk, the velocity pattern inside the dust ring is distorted.

No significant difference is found between the high velocity dispersion position (above 2 km/s) in the moment 2 map (velocity dispersion) shown in figure 2.8b and the point source position in the range of the beam size. I consider that the central dust component is co-spatial with the central star, but the spatial resolution may be insufficient to discuss the overlap between the velocity dispersion map and the central dust component. The high velocity dispersion ($\approx 3 \text{ km/s}$) along the minor axis of the disk is seen from the moment 2 map, but this component is derived from a low SN emission ($< 4\sigma$); whether this is real is unclear.

Figure 2.9a shows the position-velocity diagram (PV diagram) of ^{12}CO emission along the major axis of the ring. Assuming a stellar mass of $0.78 M_{\odot}$ (Manara et al. 2017) and a disk inclination of 51.1° , as derived from the ellipse fitting to the continuum emission (section 2.3.1),

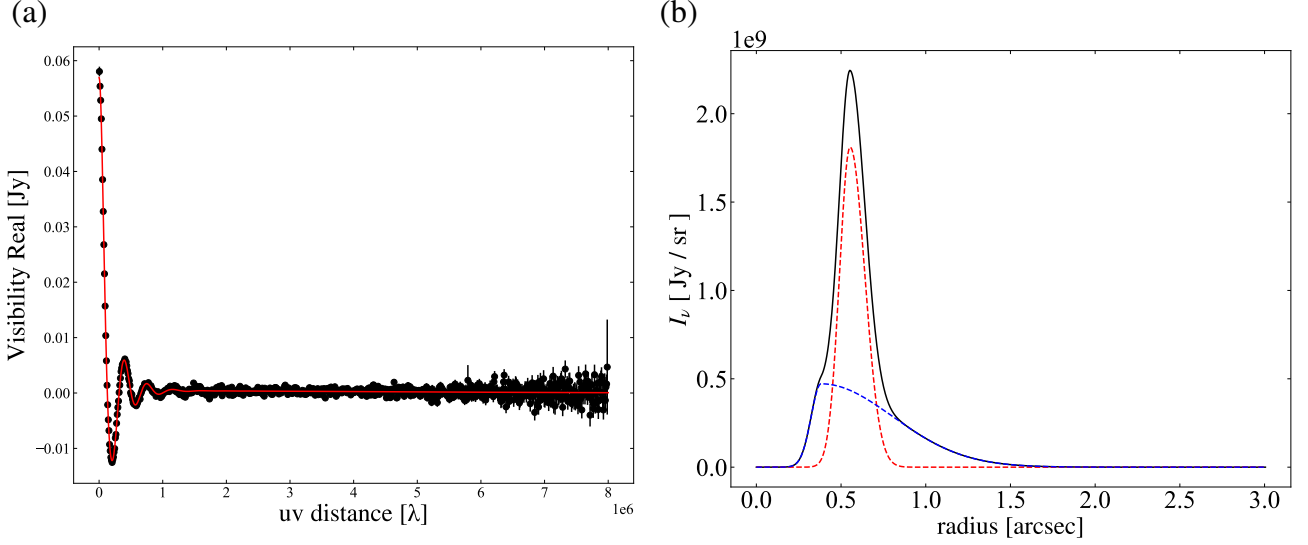


Figure 2.11: (a) Real part of the deprojected visibility for the observations (black dots) and best-fit model (red curve). (b) Radial profile of intensity as Fourier transform of the best-fit visibility (black line). The model consisted of two Gaussian ring components (blue and red dashed lines).

the Keplerian rotation curves are shown by red curves in figure 2.9a. The velocity field of the ^{12}CO emission is almost consistent with the Keplerian motion of the gas. However, there is another velocity gradient in the PV diagram along the minor axis (figure 2.9b) that cannot be accounted for by the pure Keplerian motion in the disk, and there are high-velocity components with a velocity shift of 2–4 km/s from the systemic velocity in the vicinity (<0.1 arcsec) of the central star. The distortion of the velocity pattern observed in moment 1 map may be caused by a warp (Casassus et al. 2015; Mayama et al. 2018) of the inner disk or fast radial flow (Rosenfeld et al. 2014). I discuss the nature of the velocity pattern in detail in section 2.4.

As shown in figures 2.19–2.21 of appendix 2.1, I obtained the channel maps of each CO isotopologue emission. Figure 2.10 shows the butterfly patterns in $\Delta V = -1.4 - 1.4$ km/s as red contours, corresponding to the 10σ level of the $^{12}\text{CO}(2-1)$ ($1\sigma = 1.34$ mJy/beam). As mentioned in section 2.3.1, the east side corresponds to the far side, where the butterfly pattern can be identified more clearly, when the emissions from both upper and lower disk surfaces are observed (de Gregorio-Monsalvo et al. 2013; Tsukagoshi et al. 2019).

2.3.2 Analysis of dust continuum emission in visibility domain

In this section, I investigate the asymmetry of the dust emission rings in the deprojected visibility domain. The Fourier transform of the azimuthally symmetric distribution becomes a Hankel transform (Baddour 2009)

$$V_{\text{real}}(u) = 2\pi \int_0^\infty I(r) J_0(2\pi ur) r dr, \quad (2.1)$$

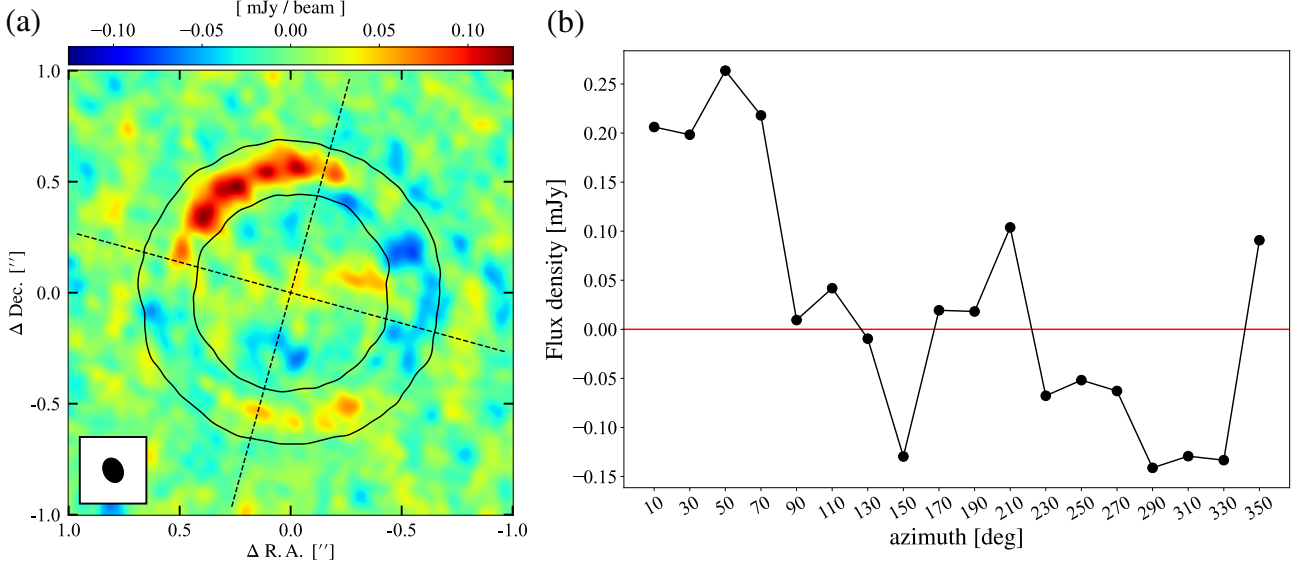


Figure 2.12: (a): Residual map obtained by subtracting the model visibility from the deprojected visibility. The synthesized beam ($0.114'' \times 0.088''$) is shown at the bottom left. The white contours denote the 20σ , where the 1σ level is the RMS noise ($= 13.8 \mu\text{Jy beam}^{-1}$). From the figure, there is a quadrupole pattern of asymmetric intensity on the outer ring. (b): Azimuth profile of flux density measured in the range of $\Delta\text{PA} = 20^\circ$ and the radius of 0.3–1 arcsec in the residual map.

where $V_{\text{real}}(u)$, $I(r)$, and J_0 are the visibility real part of the deprojected uv-distance u , intensity of the distance from the center r , and zeroth-order Bessel function of the first type, respectively.

To fit the bright ridge of the ring as well as the extended periphery, I adopted a function composed of two radially asymmetric Gaussians as the model for radial brightness distribution, as follows:

$$I(r) = \sum_{i=1}^2 I_{\text{ri,peak}} \exp \left[-\ln 2 \frac{(r - R_{\text{ri}})^2}{\text{HWHM}_{\text{ri}}^2} \right], \quad (2.2)$$

$$\text{HWHM}_{\text{ri}} = \begin{cases} \text{HWHM}_{\text{in,ri}} & (r < R_{\text{ri}}), \\ \text{HWHM}_{\text{out,ri}} & (r \geq R_{\text{ri}}), \end{cases} \quad (2.3)$$

$$I_{\text{ri,peak}} = F_{\text{tot,ri}} \left[2\pi \int_0^\infty \exp \left[-\ln 2 \frac{(r - R_{\text{ri}})^2}{\text{HWHM}_{\text{ri}}^2} \right] J_0(0) r dr \right]^{-1}, \quad (2.4)$$

where $F_{\text{tot,ri}}$ is the total flux density of the i -th ring. I determine eight parameters (R_{r1} , $\text{HWHM}_{\text{in,r1}}$, $\text{HWHM}_{\text{out,r1}}$, $F_{\text{tot,r1}}$, R_{r2} , $\text{HWHM}_{\text{in,r2}}$, $\text{HWHM}_{\text{out,r2}}$, $F_{\text{tot,r2}}$) that best fit the observed visibility profile by using **emcee** (Foreman-Mackey et al. 2013). The best-fit parameters are summarized in table 2.7 and the visibility profile for the best-fit model is shown in figure 2.11a. Figure 2.11b presents the radial brightness distribution expressed by equation (2.2).

Figure 2.12a shows the image subtracting the symmetric component from the original data. The residual asymmetric components may be characterized by a quadrupole pattern: positive residuals in the northeast and southwest to the star and negative residuals in the southeast and northwest. Figure 2.12b shows the azimuth profile of flux density measured in the range of

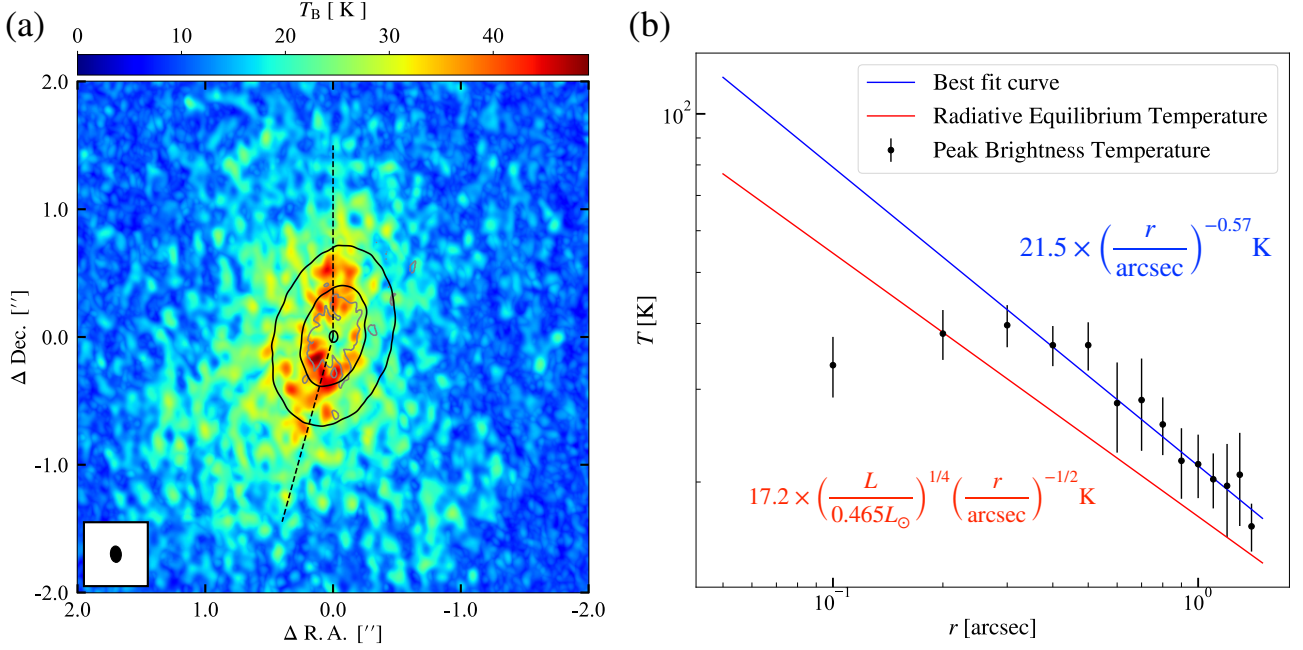


Figure 2.13: (a): Peak brightness temperature of $^{12}\text{CO}(2-1)$ emission. The synthesized beam size is shown at the bottom left. The black contours indicate the 20σ level of the dust continuum, where $1\sigma = 14.9 \mu\text{Jy}/\text{beam}$. The gray contours indicate the velocity dispersion of 0.7 km/s. (b) Radial profile of the peak brightness temperature along the straight line in (a). The blue line indicates the curve fitted in area other than gray. The red line indicates the radiative equilibrium temperature of a blackbody.

$\Delta\text{PA} = 20^\circ$ and the radius of 0.3–1 arcsec in the residual map. This pattern shows east-west asymmetry, in which the east side appears brighter than the west side on the whole.

Table 2.7: Fitting parameters of two-Gaussian model

Parameter	Value
R_{r1} [arcsec, au]	0.553, 100
$\text{HWHM}_{\text{in},r1}$ [arcsec, au]	0.075, 13.5
$\text{HWHM}_{\text{out},r1}$ [arcsec, au]	0.101, 18.3
$F_{\text{tot},r1}$ [mJy]	28.5
R_{r2} [arcsec, au]	0.385, 69.5
$\text{HWHM}_{\text{in},r2}$ [arcsec, au]	0.071, 12.9
$\text{HWHM}_{\text{out},r2}$ [arcsec, au]	0.498, 89.9
$F_{\text{tot},r2}$ [Jy]	28.0
reduced χ^2	1.14

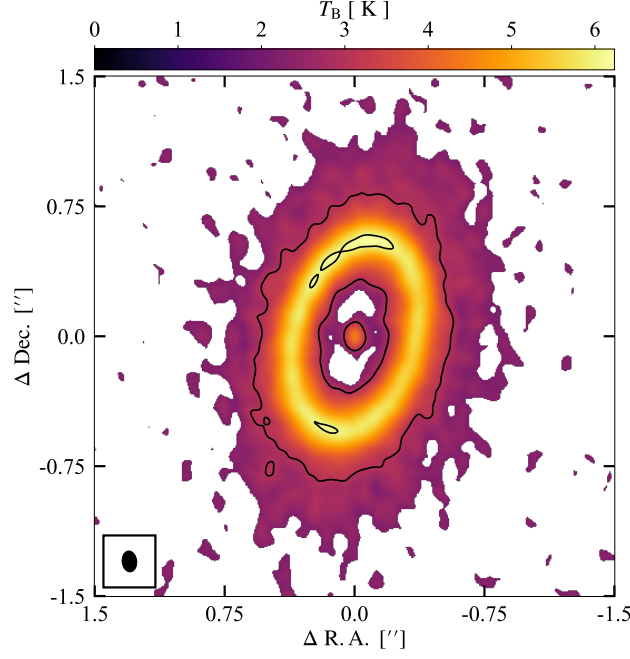


Figure 2.14: Brightness temperature of the dust. The black ellipse at the bottom left shows the synthesized beam of the dust continuum image. The contour indicates 3 K and 6 K. The area where the emission is shown to be less than 3σ in figure 2.2a is masked, where $1\sigma = 14.9 \mu\text{Jy}/\text{beam}$. The brightness temperature indicates the lower limit of the dust temperature.

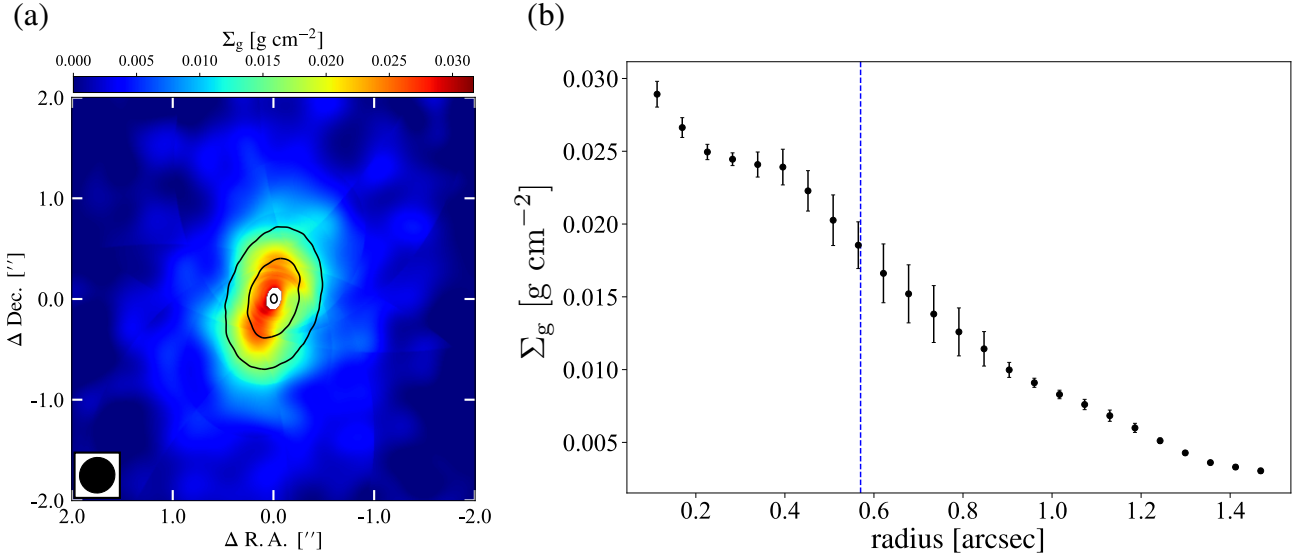


Figure 2.15: (a): Gas column density Σ_g under the assumptions of local thermal equilibrium (LTE) and the temperature distribution expressed by equation (2.5). The synthesized beam size is shown at the bottom left. (b) Radial profile of the gas column density along the major axis in (a), which was averaged with each side. The error bar is the standard deviation of the mean within the range of ± 10 mas. The blue dashed lines indicate the peak position of the dust ring.

2.3.3 Physical properties of the disk

Dust Column Density

From the continuum emission, I estimate the optical depth and column density of dust in the SY Cha system. Here, I show simple estimates based on observed quantities because I only have Band 6 data. I estimate the optical depth of the dust ring τ_d by

$$I_\nu = B_\nu(T)(1 - \exp(-\tau_d)), \quad (2.5)$$

where I_ν is the surface brightness of the continuum, and $B_\nu(T)$ is the Planck function at temperature T . As shown in figure 2.13, I adopt a temperature distribution derived from the power-law fit to the peak brightness temperature of the $^{12}\text{CO}(2-1)$, as

$$T = 415.8 \left(\frac{r}{\text{au}} \right)^{-0.57} \text{ K}. \quad (2.6)$$

Here, I fit the peak brightness temperature only in the region where the velocity dispersion of the ^{12}CO cube is less than 0.7 km/s (i.e., the channel width). The temperature near the disk mid-plane, where most of the dust particles are distributed, is expected to be lower than that in the upper layer of the disk traced by the ^{12}CO line emission (Chiang & Goldreich 1997). Therefore, the temperature expressed in equation (2.6) is the upper limit of the disk temperature. This temperature is 1.8 times higher than the radiative equilibrium temperature for a blackbody, which is as follows in the case of SY Cha,

$$T_{\text{rt}} = 231.22 \left(\frac{L}{0.465L_\odot} \right)^{\frac{1}{4}} \left(\frac{r}{\text{au}} \right)^{-\frac{1}{2}} \text{ K}, \quad (2.7)$$

where $0.465L_\odot$ is the luminosity of SY Cha (table 2.1).

The brightness temperature of the dust emission (figure 2.14) can be lower than that of equation (2.5), due to beam dilution effects and dust scattering (Liu 2019). The dust scattering depends on the dust compositions (Tazaki et al. 2016) and makes the dust emission dimmer (Soon et al. 2019; Ueda et al. 2020). However, high-spatial-resolution data for SY Cha have been taken only in Band 6 presented in this chapter. Thus, I have ignored these effects and estimated the lower limit of the optical depth of the dust ring τ_d at 225 GHz.

Assuming that the opacity of unit dust mass is

$$\kappa_d = 10 \left(\frac{\nu}{10^{12}\text{Hz}} \right)^\beta \text{ cm}^2 \text{ g}^{-1} \quad (2.8)$$

(Beckwith et al. 1990), I estimated the dust column density $\Sigma_d (= \tau_d / \kappa_d)$. At 225 GHz, κ_d derived from equation (2.8) is $2.25 \text{ cm}^2/\text{g}$ under the assumption that $\beta = 1$. The total dust mass integrated over the entire disk was estimated to be $\sim 1.0 \times 10^{-4} M_\odot$. This value may be regarded as the lower limit of the total dust because the assumed temperature is the upper limit of the disk temperature.

The lower limit of the total dust mass of the central compact ring is $\approx 0.05 M_{\oplus}$ ($\approx 1.7 \times 10^{-7} M_{\odot}$). This suggests that the inner ring might have a similar mass and orbit to Mercury in our solar system. In addition, there is no remarkable excess emission at $\lambda \lesssim 3 \mu\text{m}$ in the SED of SY Cha, as described in section 2.1. This indicates that the disk does not harbor hot ($\gtrsim 1000$ K) dust particles within at least ~ 0.1 au from the central star (see equation (2.5)). The upper limit of the size of the central ring is ~ 3.5 au since the beam size is ~ 7 au. Although the central point source is not spatially resolved even with the highest-spatial-resolution image, I consider that the central compact source has a ring-like structure as indicated in the case of DM Tau (Kudo et al. 2018).

The pressure scale height h is defined by c_s/Ω_K , where c_s and Ω_K are the sound speed and angular velocity of the Keplerian rotation, respectively. The thickness of gas can be estimated as $2h_d = 25$ au at a radius of 100 au when the temperature distribution is expressed by equation (2.5). Because the mm-size dust is sometimes concentrated near the disk mid-plane, this estimate provides the upper limit of the thickness of the dust ring. However, if the dust particles are well mixed with gas, the thickness is comparable to the ring radial width of 32 au (table 2.7), suggesting that the outer ring has a torus-like shape. Such geometry has also been observed in the Sz 91 system (Tsukagoshi et al. 2019).

Gas column density

I estimated the gas column density using the results of the $^{13}\text{CO}(2-1)$ emission, assuming local thermal equilibrium (LTE). I first derived the optical depth of the gas τ_g for each channel map by

$$I_g = B_{\nu}(T_g)(1 - \exp(-\tau_g)), \quad (2.9)$$

where T_g is the adopted gas temperature, expressed by equation (2.5). The optical depth τ_g at each position of a channel map is integrated along the velocity axis, and then converted into a column density of ^{13}CO , as described in detail in appendix 2.2. Figure 2.15a is the map of Σ_g under the assumption of the abundance ratio of $^{13}\text{CO}/\text{H}_2 = 1/67 \times 10^{-4}$ (Qi et al. 2011). Figure 2.15b shows the radial profile of the gas column density along the major axis. The peak of the gas column density (≈ 0.4 arcsec) may be located inside the dust ring. The total gas mass is estimated to be $2.2 \times 10^{-4} M_{\odot}$. The total integrated flux of C^{18}O corresponds to the total gas mass of $\sim 3.7 \times 10^{-4} M_{\odot}$ assuming the C^{18}O abundance of $\text{C}^{18}\text{O}/\text{H}_2 = 1/444 \times 10^{-4}$ (Qi et al. 2011), which is consistent with the gas mass estimated from ^{13}CO emission within a factor of 2. The abundance of CO in protoplanetary disks may be lower than the corresponding interstellar medium (ISM) values by two orders of magnitude (Zhang et al. 2020) and therefore the gas mass may be underestimated by a factor of 10–100.

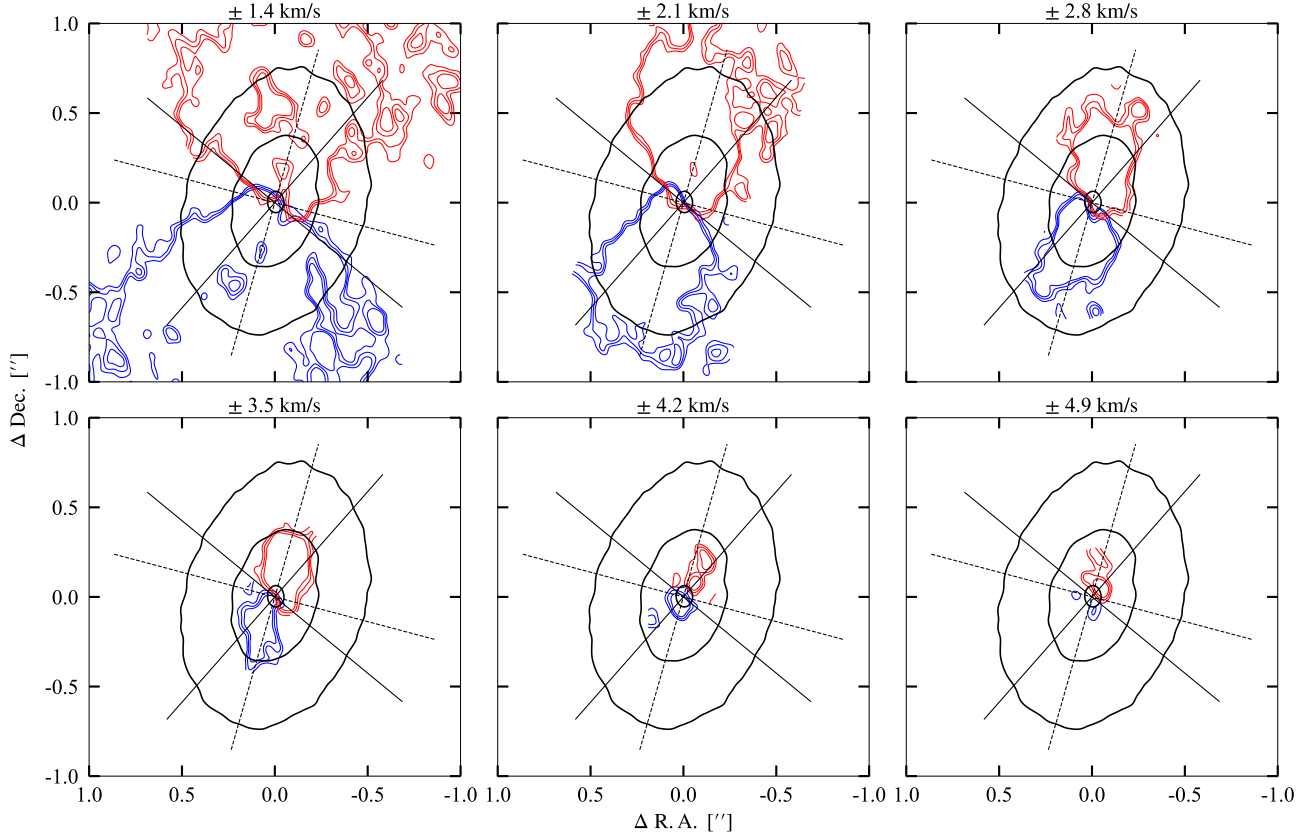


Figure 2.16: Channel map of the $^{12}\text{CO}(2-1)$ emission imaged at a channel width of 0.7 km/s. The velocities based on the systemic velocity ($V_{\text{LSRK}} = 4.1$ km/s) are labeled at the top of each plot. The red and blue contours denote the red-shifted and blue-shifted emissions, respectively. The contour levels are $[3, 6, 9]\sigma$, where the 1σ level is 1.34 mJy/beam. The dashed lines represent the major and minor axes of the outer dust disk (PA = -15.4°). The solid lines are the major and minor axes of the inner dust disk (PA = -40°).

2.4 Discussion

I detected the distortion of the line-of-sight velocity pattern that cannot be explained by the Keplerian rotation of a geometrically thin disk. As shown in figure 2.8a, the iso-velocity contour close to the center deviates from the minor axis of the ring by $\sim 25^\circ$. Figure 2.16 shows a part of the channel maps of $^{12}\text{CO}(2-1)$ emission. Each panel compares the distribution of components blue-shifted and red-shifted by the same amount from the systemic velocity. The distortion of the velocity pattern can be observed in all the panels up to $\Delta V = \pm 4.2$ km/s, and the direction of the velocity gradient appears to change with velocity.

Other transition disk systems show velocity distortion in the vicinity of the central star (e.g., Casassus et al. 2015; Mayama et al. 2018). There are two possible mechanisms causing similar velocity distortion patterns: radial flow and warped inner disk (e.g., Rosenfeld et al. 2014). Here, I discuss these two possibilities for the SY Cha system.

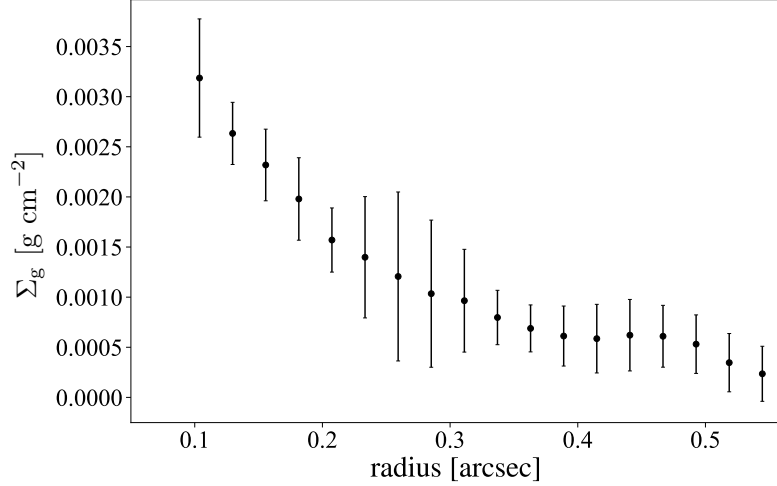


Figure 2.17: Radial profile of the gas column density calculated from ^{12}CO along the minor axis, which was averaged with each side. The error bar is the standard deviation of the mean within ± 5 mas.

2.4.1 Radial flow

In this sub-section, I discuss the possibility of radial flows in the disk. In the vicinity of the central star, the line-of-sight velocity is blue-shifted in the east and red-shifted in the west along the minor axis of the outer ring. Because the far side of the disk is in the east (see section 2.3), this distribution indicates that the velocity vector should be oriented towards the central star. Thus, fast infall may be happening in the vicinity of the central star. As shown in the PV diagram along the minor axis of the outer disk (figure 2.9b), I confirmed the non-zero velocity components within several tens of au. The infall velocity v_{in} and the line-of-sight velocity v_{los} along the minor axis of the outer ring are related by $v_{\text{in}} = v_{\text{los}} / \sin i$, where i is the inclination of the outer ring. At a projected distance of approximately 0.1 arcsec along the minor axis of the disk, a line-of-sight velocity of $v_{\text{los}} = 2\text{--}4\text{ km/s}$ is observed. Assuming that the motion is predominantly radial, this corresponds to an infall velocity of approximately 2.5–5 km/s at a deprojected distance of ~ 30 au from the central star.

From the infall velocity, I can estimate the mass infall rate $\dot{M} = 2\pi r \Sigma_g v_{\text{in}}$, where Σ_g is the gas surface density at radius r . For the estimation of Σ_g , I can employ $^{12}\text{CO}(2\text{--}1)$ emission. Because $^{12}\text{CO}(2\text{--}1)$ emission is most likely optically thick, this estimate provides the lower limit of the gas surface density, and thus the lower limit of the mass infall rate. Figure 2.17 shows the radial distribution of gas surface density derived from $^{12}\text{CO}(2\text{--}1)$ assuming the temperature expressed by equation (2.5) and the gas surface density at ~ 0.1 arcsec from the central star is $\sim 0.003 \text{ g/cm}^{-2}$ for the standard CO/H_2 ratio of 10^{-4} . This is significantly lower than the gas surface density using ^{13}CO data (figure 2.15), and the actual gas surface density can even be higher if CO is depleted (see sub-section 2.3.3). However, this gas surface density results in an estimate of the mass infall rate of $\sim 10^{-8} M_{\odot}/\text{yr}$ at $r \sim 30$ au from the central star, which is considerably higher than the stellar mass accretion rate of $\sim 3.89 \times 10^{-10} M_{\odot}/\text{yr}$ derived from

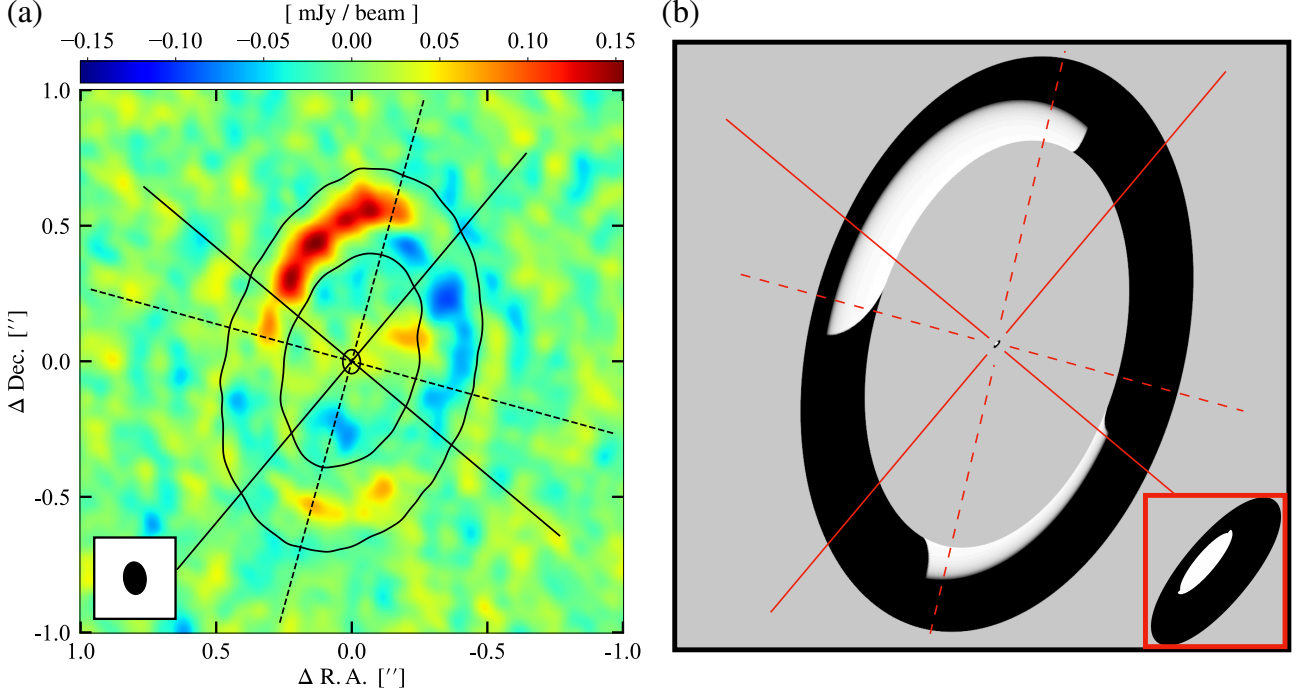


Figure 2.18: (a): Residual map projected figure 2.12a. The synthesized beam size ($0.146'' \times 0.110''$) is shown at the bottom left. The black contour indicates the 20σ level of the dust continuum, where $1\sigma = 14.9 \mu\text{Jy}/\text{beam}$. The dashed lines show the major and minor axes of the outer ring. The solid lines show the major and minor axes of the inner disk. (b): Schematic diagram of the torus model that has an inner ring (PA= -40° , inc.= 75°) and the outer ring (PA= -15.4° , inc.= 51.1°), setting the radius ratio between the outer and the inner torus to 102:3, and setting the scale height of the two rings relative to their radius to 0.2. As with (a), the red lines show the major and minor axes of the inner ring shown at the bottom-right corner, and the red dashed lines show those of the outer ring.

VLT/X-shooter spectra (Manara et al. 2017).

The evolution of protoplanetary disks is significantly affected by magnetic flux distribution (Tsukamoto et al. 2015a; Tsukamoto et al. 2015b; Bai & Stone 2017), and the wind-driven accretion by magnetohydrodynamic (MHD) effects play an important role in the mass loss process and angular momentum transport (Bai & Stone 2013; Suzuki et al. 2016). One possible scenario that explains the velocity distortion caused by radial flow while maintaining a low stellar mass accretion rate is the combination of fast accretion only in the upper layer of the disk and disk wind. The mass accretion rate is low in the close vicinity of the star, probably because the accreting materials in the upper layer of the disk are loaded on magnetic field lines, and are disposed as disk wind (e.g., Sheikhnezami et al. 2012). However, the wind mass loss rate is probably a few times the stellar mass accretion rate at most (Hasegawa et al. 2021), and therefore, it may be difficult to load almost all the accreting material from the outer disk onto the disk wind.

2.4.2 Warp

If the system has a warp structure, I expected the existence of a misaligned inner dust disk (Casassus et al. 2018). In this sub-section, I assumed that the disk system has a misalignment between the inner and outer rings. The PA of the disk major axis should be along the direction with the highest line-of-sight velocity gradient; therefore, I assumed that the major axis of the inner ring is along the PA of -40° (shown in figure 2.8a).

Figure 2.18a shows the residual intensity map shown in figure 2.12a projected onto the plane of the sky using the outer ring inclination (51.1°) and position angle (-15.4°). As shown in the figure, the residual pattern is brightest in the northeast direction, which is similar to the direction along the minor axis of the inner disk, as suggested by the velocity gradient. A misaligned inner disk can cast a shadow onto the outer disk, causing azimuthally asymmetric illumination (Marino et al. 2015). This asymmetric illumination may cause temperature variations on the ring, thereby causing asymmetry of the dust continuum emission. Our observations show that the degree of azimuthal asymmetry along the outer ring is $\sim 15\%$. This suggests that the difference in temperature in the bright and dark regions is a few Kelvin, while the average temperature of the outer ring is ~ 30 K. This temperature difference may be caused by the asymmetric illumination pattern caused by the inner disk.

Here, I investigate the shadow cast by the inner disk using a simplified model made with the 3D graphics software Blender. I considered two optically thick misaligned rings sharing the same center where a point light source is placed (figure 2.18b). The outer ring has the same position angle and inclination as observed (PA $= -15.4^\circ$, inc. $= 51.1^\circ$), while the inner ring has a position angle of -40° , which is indicated by the observed velocity distortion. Thereafter, I varied the inclination of the inner ring to investigate the changes in the illumination pattern on the outer disk. I set the radius ratio between the outer and inner torus to 102:3 and the scale height of the two rings relative to their radius to be 0.2. I found an illumination pattern similar to figure 2.18a when the inner disk has an inclination of $70-90^\circ$ (figure 2.18b). The northeast part of the outer ring is illuminated, and the temperature of this part may be higher than that of the other parts. Moreover, the inner disk with an inclination of $70-90^\circ$ is consistent with the observed velocity distortion of a few km/s, because the Keplerian velocity at $r \sim 0.1$ arcsec ≈ 20 au along the major axis of the inner disk is 6 km/s.

Therefore, the misaligned inner disk is a plausible interpretation of the velocity distribution and dust asymmetry. To obtain a realistic temperature distribution, rather than an illumination pattern, I need full radiative transfer modeling, which will be presented in a future work.

Detailed study of the warped inner disk is left as a future work. As one of the approach, I propose conducting an observation of ro-vibrational CO line emission at $4.7 \mu\text{m}$ for SY Cha. This line is formed in the heated surfaces of the disk at radii of 0.1–10 au (Brown et al. 2013), and the compact inner gas disk thus is thus expected to be traced.

Pontoppidan et al. (2008) observed a CO $4.7 \mu\text{m}$ line in protoplanetary disks around SR 21, HD 135344B and TW Hya using the CRIRES high-resolution infrared spectrometer on the

Very Large Telescope (VLT) and estimated the PA and inclination of the inner disks to within a few degrees, though with a caveat that these estimates listed by Pontoppidan et al. (2008) rely on accurate estimation of the stellar mass, which was not well constrained at the time.

2.5 Conclusions

I reported ALMA observations of the transition disk around SY Cha. The dust continuum emission and the $J=2-1$ line emission of CO isotopologues at Band 6 were observed. My conclusions are summarized as follows;

1. The 225 GHz dust continuum emission shows a clear ring structure with a radius of ~ 100 au and central point sources. The total dust mass is estimated to be $1.0 \times 10^{-4} M_{\odot}$. I suggested that the central point source is actually a ring because the SED does not show a significant IR excess. From visibility-domain analyses, I found extended emission outside the outer ring and weak asymmetry in the outer ring. The asymmetry shows that the ring is bright in the north-east direction.
2. I detected ^{12}CO , ^{13}CO and C^{18}O emission. The ^{13}CO emission is concentrated in the vicinity of the dust ring, while ^{12}CO emission was detected inside the dust ring, indicating that there is some gas remaining inside the dust ring. Assuming that ^{12}CO is a tracer of disk temperature, I estimated the total gas mass as $2.2 \times 10^{-4} M_{\odot}$ and the peak column density of the disk gas as $\sim 0.025 \text{ g/cm}^2$ if CO is not depleted. The estimated gas value may be increased by a factor of 10–100 by CO depletion.
3. The moment 1 map of the CO line emission shows that the velocity pattern agree with the Keplerian rotation around $0.78 M_{\odot}$; however, the velocity pattern in the vicinity of the central star is distorted. The origin of the velocity distortion is yet inconclusive. If the velocity distortion is caused by radial flow, the mass inflow rate at ~ 30 au is well above the stellar mass accretion rate, indicating that a significant amount of mass is being ejected as disk wind. If the velocity distortion is caused by a warped inner disk, I expected that the inner disk is close to edge-on (inclination of $70-90^{\circ}$). The weak asymmetry of the outer disk is consistent with the asymmetric illumination of the central star caused by the warped inner disk.

These findings provide concrete observational evidence that the inner disk of SY Cha is not only present but potentially misaligned, influencing the outer disk dynamically and thermally. In particular, the observed gas kinematics and dust continuum asymmetry suggest altered irradiation conditions in the outer disk, which may lead to localized thermal effects such as shadow-induced cooling.

To further interpret these features and constrain the spatial structure of the misaligned inner disk, a three-dimensional analysis of the shadow geometry is required. In the next chapter, I

investigate the relationship between the inclination of the inner disk and the resulting shadow and temperature structure in the outer disk.

Software: *astropy*, *CASA*, *Numpy*, *Matplotlib*, *emcee*, *keplerian_mask*, *bettermoments*, *Blender*

Acknowledgement

This study makes use of the following ALMA data: ADS/JAO.ALMA#2018.1.00689.S. ALMA is a partnership of ESO (representing its member states), NSF (USA) and NINS (Japan), together with NRC (Canada), MOST and ASIAA (Taiwan), and KASI (Republic of Korea), in cooperation with the Republic of Chile. The Joint ALMA Observatory is operated by ESO, AUI/NRAO and NAOJ. The National Radio Astronomy Observatory is a facility of the National Science Foundation, operated under cooperative agreement by Associated Universities, Inc. This work was supported by JSPS KAKENHI grant numbers 17H01103 and 18H05441. This paper makes use of the following ALMA data: ADS/JAO.ALMA#2018.1.00689.S. ALMA is a partnership of ESO (representing its member states), NSF (USA) and NINS (Japan), together with NRC (Canada), MOST and ASIAA (Taiwan), and KASI (Republic of Korea), in cooperation with the Republic of Chile. The Joint ALMA Observatory is operated by ESO, AUI/NRAO and NAOJ. Y.Y. was supported by the NAOJ ALMA Scientific Research grant number 2019-12A. Y.H. was supported by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration (80NM0018D0004). T.T. was supported by JSPS KAKENHI grant number 20K04017.

Appendix 2.1: Channel maps

Figures 2.19–2.21 show the channel maps of the CO line emission at $V_{\text{LSRK}} = -4.3$ to $+12.5$ km/s. See table 2.4 for information of these channel maps.

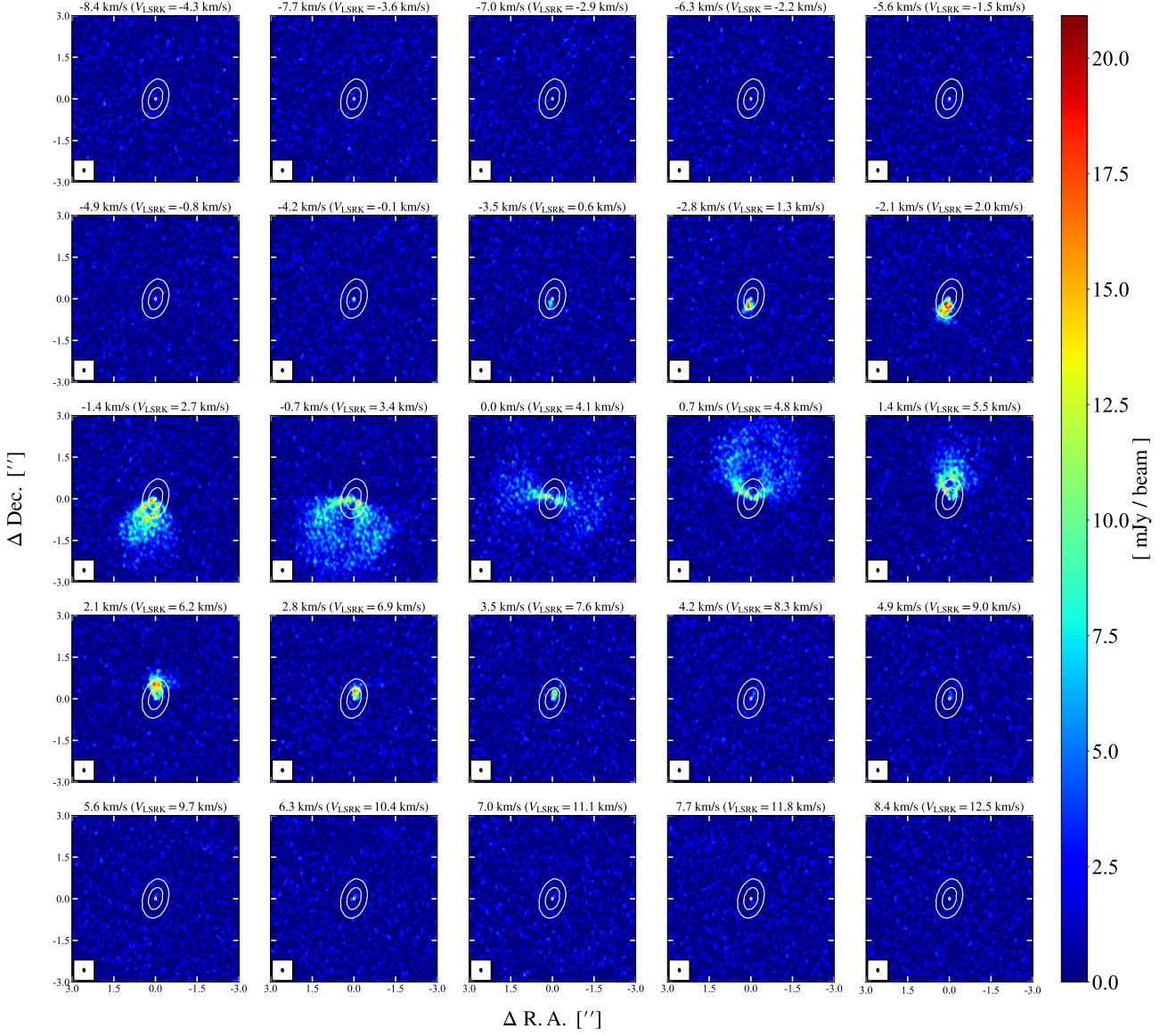


Figure 2.19: $^{12}\text{CO}(2-1)$ Channel Map in the concatenated data. The black ellipse at the bottom left of each panel represents the beam size. The contours in these maps indicate the 20σ level of the dust continuum, where $1\sigma = 14.9 \mu\text{Jy}/\text{beam}$.

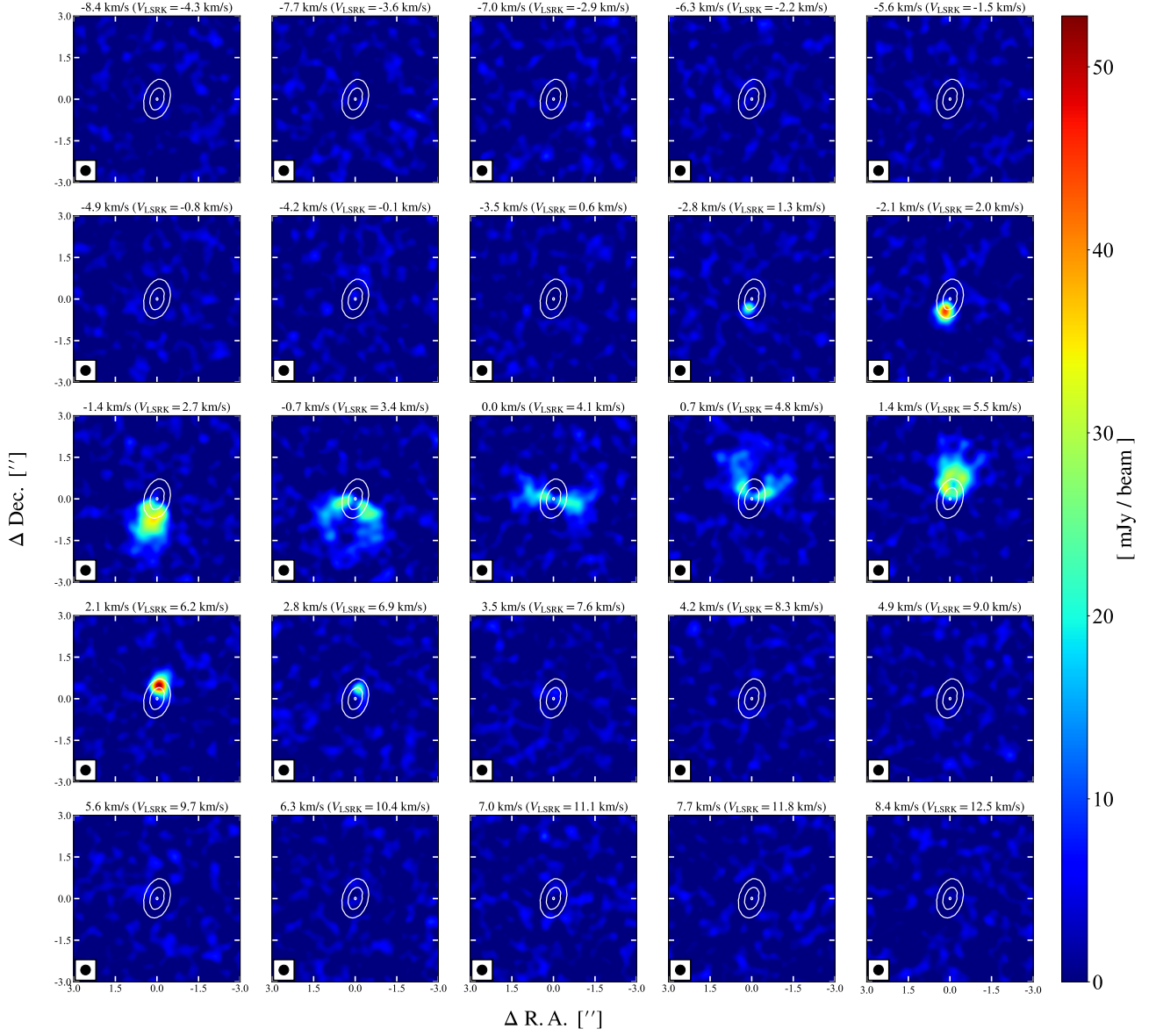


Figure 2.20: $^{13}\text{CO}(2-1)$ Channel Map in the CC data. The black ellipse at the bottom left of each panel represents the beam size. The contours in these maps indicate the 20σ level of the dust continuum, where $1\sigma = 14.9 \mu\text{Jy/beam}$.

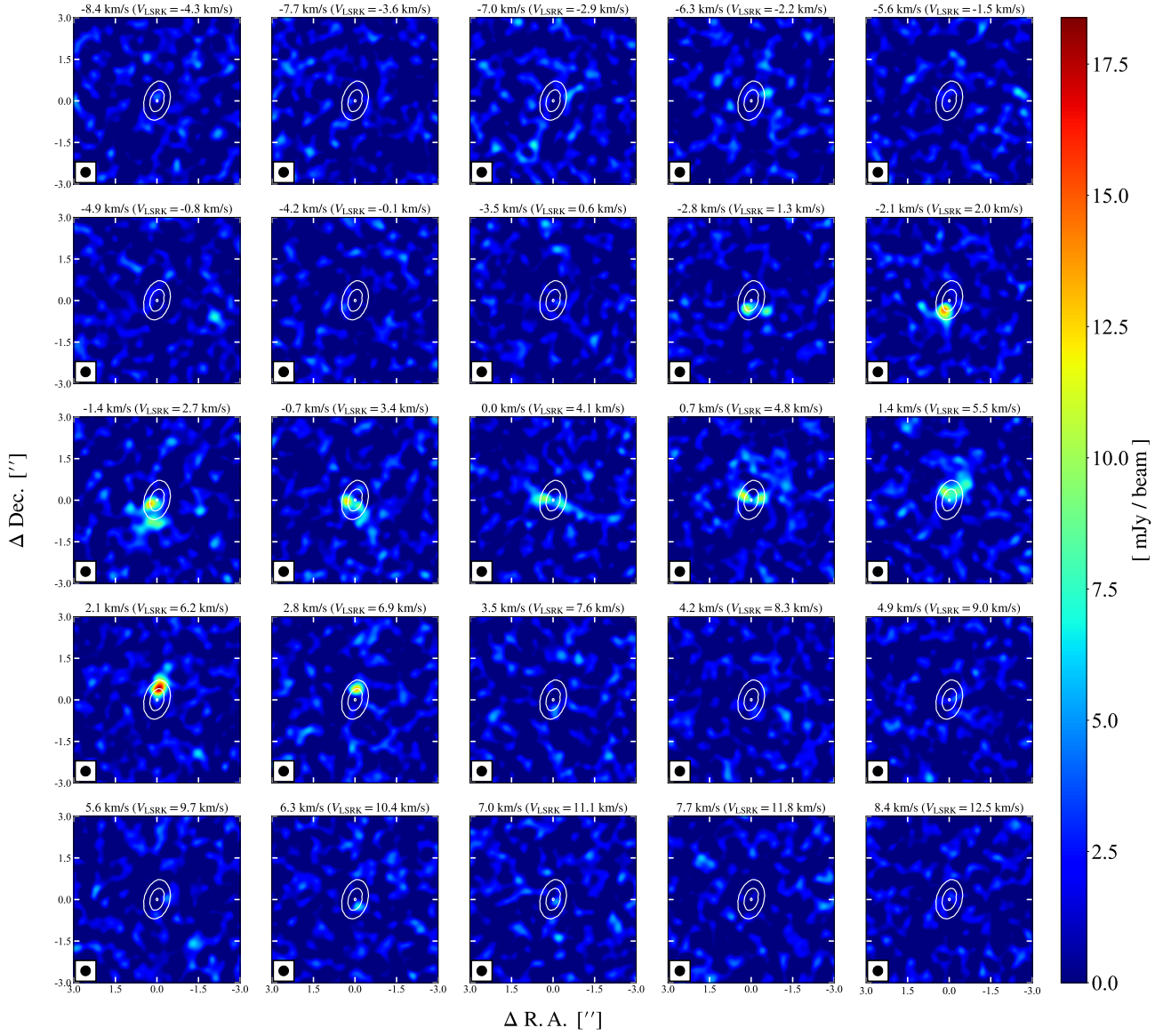


Figure 2.21: $\text{C}^{18}\text{O}(2-1)$ Channel Map in the CC data. The black ellipse at the bottom left of each panel represents the beam size. The contours in these maps indicate the 20σ level of the dust continuum, where $1\sigma = 14.9 \mu\text{Jy}/\text{beam}$.

Appendix 2.2: Calculation of molecular column density

Calculating the optical depth τ_g using equation (2.9), the molecular spectral line column density is expressed by the following equations (Mangum & Shirley 2015):

$$N_{\text{tot}} = \frac{3h}{8\pi^3|\mu_{lu}|^2} \frac{Q_{\text{rot}}}{g_u} \exp\left(\frac{E_u}{kT_{\text{ex}}}\right) \times \left[\exp\left(\frac{h\nu}{kT_{\text{ex}}}\right) - 1 \right]^{-1} \int \tau_g dv, \quad (2.10)$$

$$g_u = 2J_u + 1, \quad (2.11)$$

$$E_u = hBJ_u(J_u + 1), \quad (2.12)$$

$$Q_{\text{rot}} \approx \frac{kT_g}{hB} + \frac{1}{3}, \quad (2.13)$$

$$|\mu_{lu}|^2 = \mu^2 S, \quad (2.14)$$

$$S = \frac{J_u^2}{J_u(2J_u + 1)}, \quad (2.15)$$

where μ , J_u , and B are the dipole moment, rotational quantum number of the line emission upper level, and rigid rotor rotational constant, respectively (parameters of each CO isotopologue are shown in table 2.8). The excitation temperature T_{ex} is equal to T_g in the LTE analysis. The gas column density is then calculated by

$$\Sigma_g = \frac{m_{\text{H}_2} N_{\text{tot}}}{X}, \quad (2.16)$$

where $m_{\text{H}_2} (= 3.32 \times 10^{-24} \text{g})$ is the molecular mass of H_2 , and X is the molecular abundance ratio with H_2 .

In this study, the following equation was used to calculate the total flux of the $\text{C}^{18}\text{O}(2-1)$ line emission:

$$N_{\text{tot}}(\text{C}^{18}\text{O}) = \frac{3c^2}{16\pi^3\Omega_S S \mu^2 \nu^3} \left(\frac{Q_{\text{rot}}}{g_u} \right) \times \exp\left(\frac{E_u}{kT}\right) \int S_\nu dv, \quad (2.17)$$

where S_ν and Ω_S are the flux density and beam size, respectively.

Table 2.8: Physical parameters of CO isotopologue

	$^{12}\text{CO}(2-1)$	$^{13}\text{CO}(2-1)$	$\text{C}^{18}\text{O}(2-1)$
μ [10^{-18} esu cm]	0.110	0.110	0.1098
J_u	2	2	2
B [MHz]	57635.96	55101.01	54891.42
abundance ratio with H_2	10^{-4}	$(1/67) \times 10^{-4}$	$(1/444) \times 10^{-4}$

Appendix 2.3: Corner plots of MCMC fitting results

Figure 2.22 shows corner plot of the MCMC posteriors calculated in visibility fitting given the model presented as equation (2.2). The plot shows the posterior sampling provided by the last 4000 steps of the 200 walkers chain.

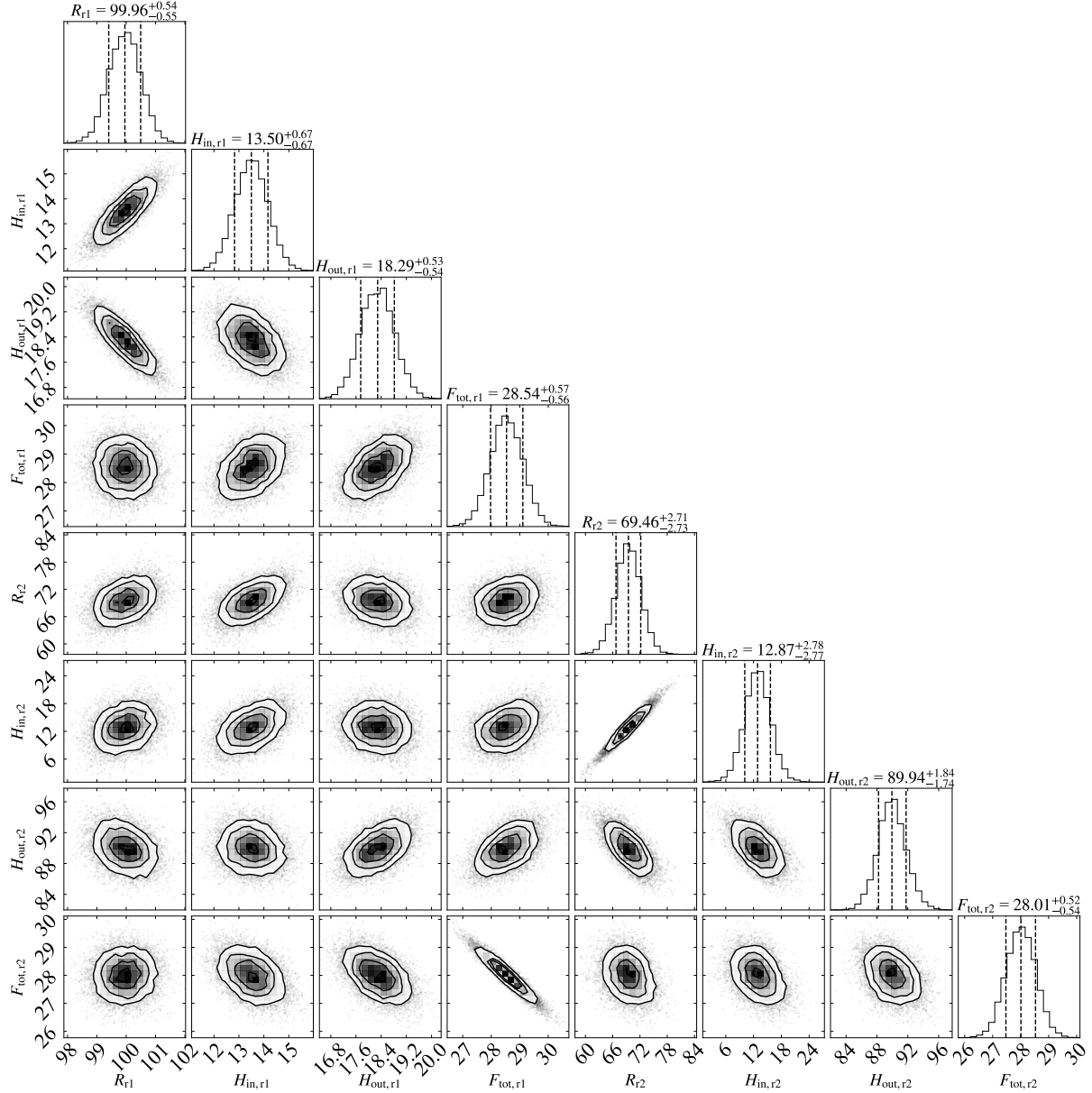


Figure 2.22: Corner plot of MCMC results in visibility analysis. The median values and 1σ of the best-fit parameters are shown as vertical dashed lines.

Appendix 2.4 : ALMA Pipeline Calibration

The ALMA science pipeline provides an automated calibration and imaging procedure which produces standard science quality data products for both radio expert and non-radio expert astronomers ([Geers et al. 2019](#)). The pipeline tasks are designed to be modular, each can be run via the CASA task interface or as part of an automated Python script. The pipeline tasks become available by starting up CASA using the command:

```
casa —pipeline
```

In general, to restore pipeline calibrated data, I have to run the `scriptForPI.py`, which is basically only compatible with CASA versions similar to the one used for its creation. The `scriptForPI.py` could run using the command:

```
execfile('scriptForPI.py')
```

Tables [2.9](#) and [2.10](#) show description of the pipeline tasks ([ALMA Science Pipeline User's Guide for CASA 5.4 2018](#); [ALMA Science Operations: Cycle 6 Pipeline Reference Manual 2018](#)).

Table 2.9: pipeline task

task name	description
<code>hifa_importdata</code>	This task loads the specified visibility data into the pipeline context unpacking and/or converting it as necessary.
<code>hifa_flagdata</code>	The data performs basic flagging operations on a list of MeasurementSets.
<code>hifa_fluxcalflag</code>	Locate and flag line regions in solar system flux calibrators. Fills the model column with the model visibilities.
<code>hif_rawflagchans</code>	Flags deviant baseline/channels in the raw data.

Table 2.10: pipeline task

task name	description
<code>hif_refant</code>	Selects a list of reference antennas and outputs them in priority order. The priority order is determined by a weighted combination of scores derived by the antenna selection heuristics.
<code>h_tsyscal</code>	Derive the Tsys calibration for list of ALMA MeasurementSets.
<code>hifa_tsysflag</code>	Flag deviant system temperatures for ALMA interferometry measurements.
<code>hifa_antpos</code>	Derive an antenna position calibration table.
<code>hifa_wvrgcalflag</code>	Generate a gain table based on Water Vapor Radiometer (WVR) data, interpolating over antennas with bad radiometers.
<code>hif_lowgainflag</code>	Flags data for antennas with unusually low or high gains.
<code>hif_setmodels</code>	Derive flux densities for point source transfer calibrators using flux models for reference calibrators.
<code>hifa_bandpassflag</code>	Compute bandpass calibration with flagging.
<code>hifa_spwphaseup</code>	Compute phase calibration spw map and per spw phase offsets.
<code>hifa_gfluxscaleflag</code>	Derive the flux density scale with flagging.
<code>hifa_gfluxscale</code>	Derive flux densities for point source transfer calibrators using flux models for reference calibrators.
<code>hifa_timegaincal</code>	Determine temporal gains from calibrator observations.
<code>hif_applycal</code>	Apply precomputed calibrations to the data.
<code>hif_makeimlist</code>	Create a list of images to be cleaned.
<code>hif_makeimages</code>	Compute a cleaned image for a particular target source/intent and spectral window.
<code>hifa_imageprecheck</code>	The representative source and spw containing the representative frequency selected by the PI in the ALMA Observing Tool (OT) are used to calculate the synthesized beam and to make sensitivity estimates for the aggregate bandwidth and representative bandwidth for several values of the robust parameter.
<code>hif_checkproductsizes</code>	Check interferometry imaging product size and try to mitigate to maximum allowed values.

Appendix 2.5 : Visibility weighting

With reference to [Thompson et al. \(2017\)](#), I explain the visibility weighting.

In radio interferometry study, the Fast Fourier transform (FFT) is often used to make image from measured visibility. The speed of the fast algorithm for the FFT is a major advantage in computing large images. At this time, a gridding is used to transform visibility values into uniformly sampled grid points. the visibility samples in each grid cell is given a weight in the imaging step. This weighting can be used to account for differences in the density of sampling in different parts of the (u, v) plane, or to improve sensitivity to extended objects. Here we briefly review some weighting schemes.

Density weighting

- **Natural weighting**

This gives constant weights to all visibilities (or, more strictly, inversely proportional to the noise variance of a visibility). This weighting gives optimum point-source sensitivity in an image. However the synthesised beam-shape and sidelobe levels are usually poor.

- **Uniform weighting**

This gives a weight inversely proportional to the sampling density. This form of weighting minimises the sidelobe level. However the noise level can be a factor of 2 worse than natural weighting.

- **Briggs weighting**

Uniform weighting minimizes sidelobes, whereas natural weighting minimizes the noise level. Briggs weighting provides a compromise between the two, doing so in an optimal sense. The weighting of cell (i, k) in the (u, v) plane whose visibility has an rms error of σ_{ik} is specified as

$$w_{ik} = \frac{1}{S^2 + \sigma_{ik}^2}, \quad (2.18)$$

$$S^2 = \frac{(5 \times 10^{-R})^2}{\bar{w}}, \quad (2.19)$$

$$\bar{w} = \frac{1}{n_c} \sum \frac{1}{\sigma_{ik}^2}, \quad (2.20)$$

where R is the robustness factor (-2 to 2), and $\bar{w}$ is the average variance weighting factor over the n_c cells in the image. $R = 2$ makes S very small with respect to $\bar{w}$ so that the weighting approaches natural weighting, whereas $R = -2$ makes S large with respect to $\bar{w}$ so that the weighting approaches the uniform weighting.

uv-taper

The uv-taper applies a multiplicative Gaussian to the spatial-frequency grid, to weight down high spatial-frequency measurements relative to the rest. This suppresses artifacts arising from poorly sampled regions near and beyond the maximum spatial frequency and significantly increases the detectability of an extended source. However, it also degrades the resolution. Gaussian weighting can be combined with any of the above weighting schemes to achieve some form of balance between sidelobes and sensitivity.

Appendix 2.6 : CLEAN

With reference to [Cornwell \(2008\)](#) and [Thompson et al. \(2017\)](#), I explain the CLEAN algorithm.

Radio synthesis arrays image the radio sky not through a single large physical aperture but by synthesising a virtual aperture of equivalent size and angular resolution. This is done by invoking the van Cittert-Zernike theorem relating the visibility and the intensity:

$$V(u, v) = \mathcal{F}[I(x, y)]. \quad (2.21)$$

For a real array, the completely sampled visibility function $V(u, v)$ is not available and we have only incomplete samples of the visibility at discrete locations in the (u, v) plane. Ignoring for the moment the effects of noise, we can represent this by replacing $V(u, v)$ by the sampled visibility function $S(u, v)V(u, v)$, where the sampling function $S(u, v)$ is:

$$S(u, v) = \sum_k w_k \delta(u - u_k) \delta(v - v_k), \quad (2.22)$$

where w_k is weight for each visibility data.

Inserting this into the Fourier inverse, we then obtain the dirty image:

$$I^D(x, y) = \mathcal{F}^{-1}[S(u, v)V(u, v)]. \quad (2.23)$$

Applying the convolution theorem from Fourier transform theory, we find that the dirty image is the convolution of the true image I with a synthesized beam B (figure 2.23):

$$I^D(x, y) = B * I(x, y), \quad (2.24)$$

where the synthesized Beam or Point Spread Function $B = \mathcal{F}^{-1}[S]$ is given by:

$$B(x, y) = \sum_k \cos(2\pi(u_k x + v_k y)) w_k. \quad (2.25)$$

Thus the deconvolution problem is to solve for I from knowledge of the dirty image I^D , and the point spread function B .

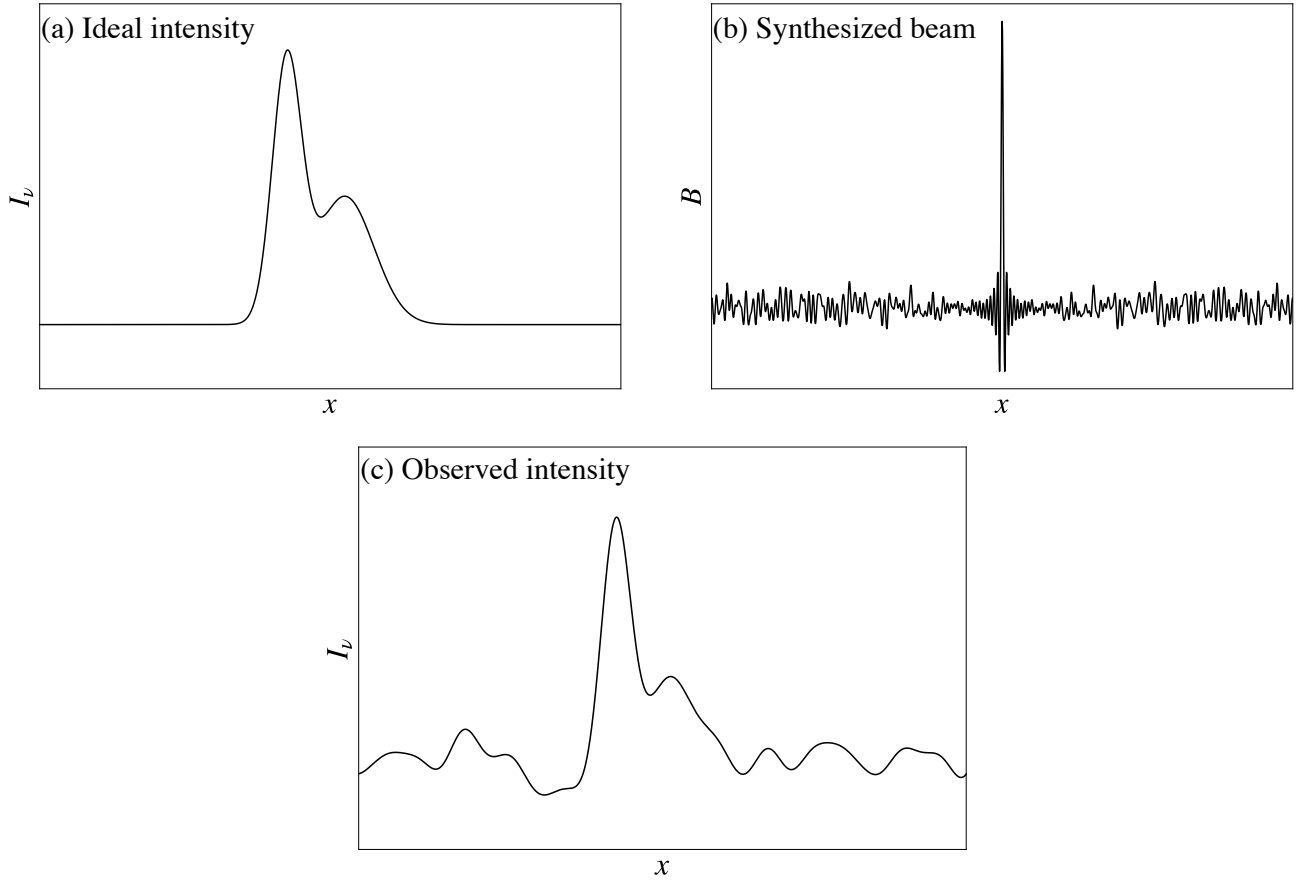


Figure 2.23: The observed intensity is interpreted as the convolution of the true intensity with a synthesized beam.

CLEAN algorithm

Högbom CLEAN is a basic numerical deconvolving process usually applied in the image domain (Högbom 1974). The CLEAN algorithm represents the image as a number of point sources in an otherwise empty field of view. A simple iterative procedure is used to find the locations and strengths of these point sources. The CLEAN algorithm looks for the brightest pixel (called a CLEAN component) in a specified region in the image (figure 2.24). Having found this peak pixel, it subtracts a fraction (called the loop gain, and generally about 0.1 or less) of the synthesized beam from the residual (dirty) image at the location of that brightest pixel. This subtracted image is called the residual image. The search and subtraction loop is repeated until the sidelobes in the image are reduced to below the noise level. Performance of the deconvolution can be improved by restricting the search to a region wherein the image brightness is expected to be non-zero.

In the end, we convolve the delta functions in the cleaned model with a CLEAN beam response B_G , that is, replace each delta function with a CLEAN beam function of corresponding amplitude (figure 2.25). The clean beam is often chosen to be a Gaussian with a half-amplitude width equal to that of the original synthesized beam. Moreover, adding the residuals into the convolved image, which is then the output of the process. When the residuals are added, the

Fourier transform of the image is equal to the measured visibilities.

The scheme for the Högbom CLEAN algorithm is presented in Algorithm 1. CLEAN is a greedy algorithm: it consists of a sequence of iterations, and within each iteration, the choice made depends only on information available at the iteration. There is no guarantee that the result of the CLEAN algorithm is globally optimal in any particular sense, although there is a conjecture that the image L1 norm is minimized (Marsh & Richardson 1987). The interpretation of Högbom CLEAN in Compressive sensing has been developed as a matching pursuit algorithm calculating residual vectors in image space (Li et al. 2011).

Algorithm 1 Högbom CLEAN

Initialize:

Model: $I^M = 0$

Residual Image: $I^R = I^D$

Repeat:

- (1) Find strength and location of peak residual
- (2) Add this component to current model, scaled by loop gain
- (3) Update residual image using precomputed terms

Until:

Either $\max(I^R) < \text{threshold}$

Or Maximum number of components identified

Finalize:

Convolve current model by clean beam: $B_G * I^M$

Add residuals to get restored image: $B_G * I^M + I^R$

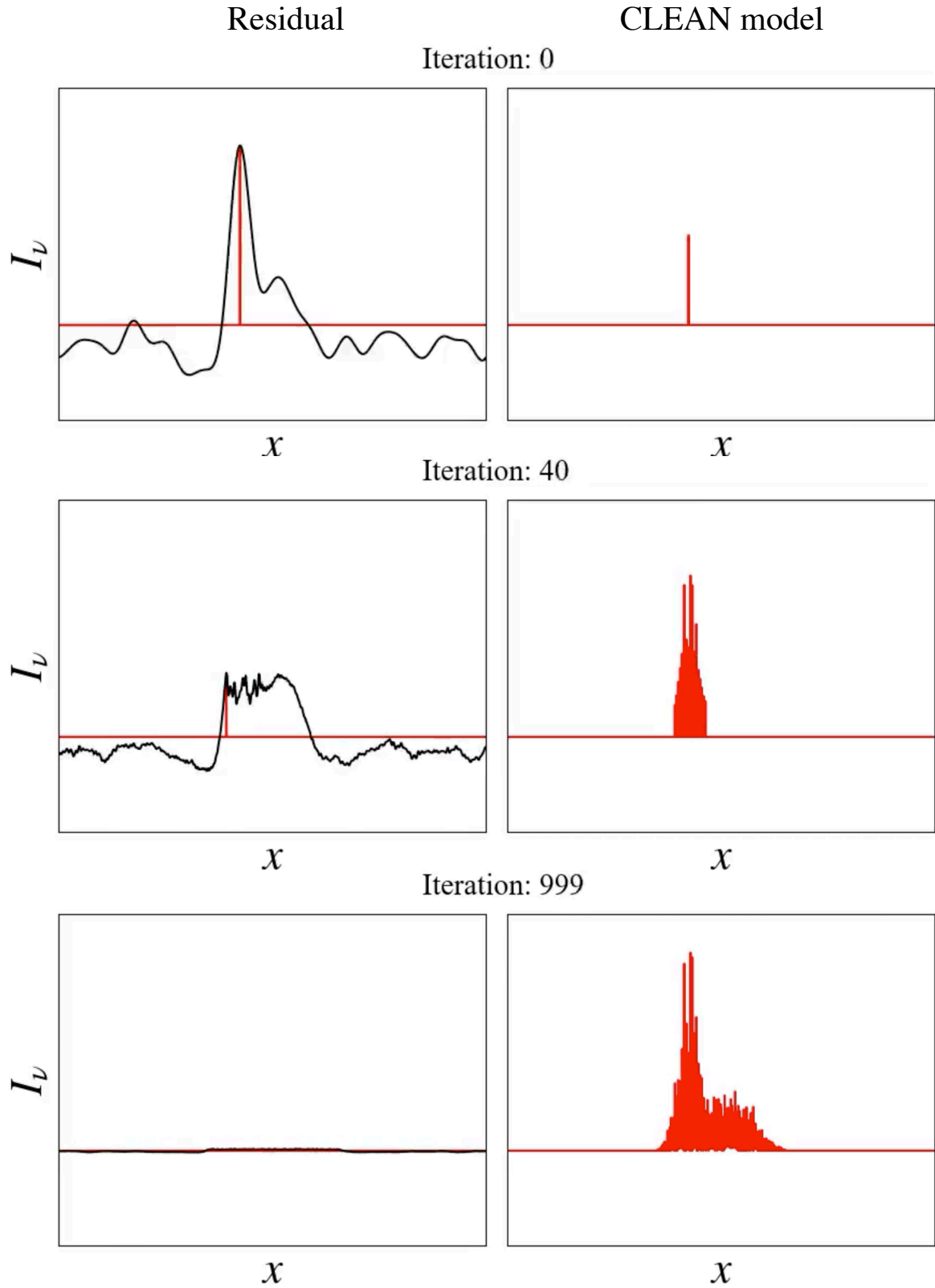


Figure 2.24: Residual and CLEAN model in each iteration. Having found this peak point, it subtracts a fraction of the synthesized beam from the residual at the location of that brightest point.

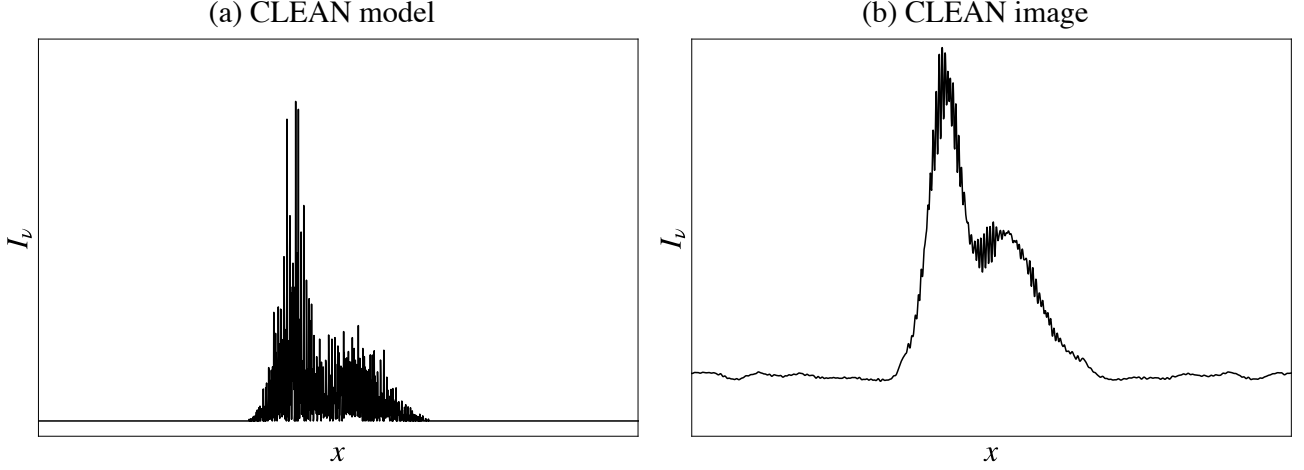


Figure 2.25: The CLEAN image (b) is outputted as the image which convolved the model (a) with CLEAN beam and added to the final residuals.

Multi-Scale CLEAN

The model is based on the assumption that the true sky brightness is a summation of appropriately scaled and centered extended components. Denoting the component width by a single parameter α_q , the model is expressed by:

$$I^M = \sum_q I_q m(x - x_q, y - y_q, \alpha_q). \quad (2.26)$$

Following the greedy strategy used in the CLEAN algorithm, we search for the n th component by looking for a peak in:

$$I^R(n) = I^D - B * I^M(n-1) \quad (2.27)$$

where the search is over I_q, x_q, y_q, α_q . In deciding which scale to select, we must apply a bias towards smaller scales. We evaluate the maximum residual for a set of scale sizes, and choose that scale size with the maximum adjusted residual.

$$S(\alpha) = 1 - 0.6 \frac{\alpha}{\alpha_{\max}} \quad (2.28)$$

In any iteration, the component selected is thus located at the peak with the appropriate scale size. The magnitude of the component is given by the loop gain times the peak. The algorithm can be made efficient by careful precomputation of all terms that are needed for the update:

$$B * m(\alpha_q), \quad B * m(\alpha_p) * m(\alpha_q) \quad (2.29)$$

[Cornwell \(2008\)](#) have chosen a tapered, truncated parabola as the component shape:

$$m(r, \alpha) = \Psi(r) \left(1 - \left(\frac{r}{\alpha} \right)^2 \right) \quad (2.30)$$

where Ψ is a prolate spheroidal wave function. The difference between this function and a suitably scaled Gaussian is quite minor. For zero α , this corresponds to a Dirac delta function.

The scheme for the MS-CLEAN algorithm is presented in Algorithm 2 (Cornwell 2008).

Algorithm 2 MS-CLEAN

Initialize:

Model: $I^M = 0$

Residual Image: $I^R = I^D$

For each scale α_q

– calculate scale-convolved residual $I_\alpha^R = m(\alpha) * I^R$

– calculate scale bias $S(\alpha) = 1 - 0.6 * \alpha / \alpha_{\max}$

For each pair of scales α_p, α_q , calculate cross term: $B * m(\alpha_p) * m(\alpha_q)$

Repeat:

(1) For each scale, find strength and location of peak residual

(2) Choose scale with maximum residual, after multiplying by scale-dependent bias terms

(3) Add this component to current model, scaled by loop gain

(4) Update all residual images using precomputed terms

Until:

Either $\max(I_\alpha^R) < \text{threshold}$

Or Maximum number of components identified

Finalize:

Convolve current model by clean beam: $B_G * I^M$

Add residuals to get restored image: $B_G * I^M + I^R$

Appendix 2.7 : Self-Calibration

Principle of self-calibration

Because the interpolated calibrator gains do not determine the antenna gains perfectly at each time stamp, the quality of the resultant image is limited. The technique of self-calibration is widely used to make additional corrections to the antenna gains as a function of time. The self-calibration procedure is often performed iteratively, each time with a better model, until the sequence converges and no more improvement in the image quality can be made.

With reference to Thompson et al. (2017), I explain the self-calibration. The procedure in self-calibration is to use a least-mean-squares method to minimize the square of the modulus of the difference between the observed visibilities, V_{ij}^{obs} , and the corresponding values for the derived model, V_{ij}^{mod} . The expression that is minimized is

$$\sum_{\text{time}} \sum_{i < j} w_{ij} |V_{ij}^{\text{obs}} - g_i g_j^* V_{ij}^{\text{mod}}|^2, \quad (2.31)$$

where the weighting coefficient w_{ij} is usually chosen to be inversely proportional to the variance of V_{ij}^{obs} , and the quantities shown are all functions of time within the observing period. Equation (2.31) can be written

$$\sum_{\text{time}} \sum_{i < j} w_{ij} |V_{ij}^{\text{mod}}|^2 |X_{ij} - g_i g_j^*|^2, \quad (2.32)$$

where

$$X_{ij} = \frac{V_{ij}^{\text{obs}}}{V_{ij}^{\text{mod}}}. \quad (2.33)$$

If the model is accurate, the ratio X_{ij} of the uncalibrated observed visibility to the visibility predicted by the model is independent of u and v but proportional to the antenna gains. Thus, the values of V_{ij} simulate the response to a calibrator and enable the gains to be determined. However, since the initial model is only approximate, the desired result must be approached by iteration.

The self-calibration procedure is:

1. Make an initial model of the source using whatever constraints we have on the source structure.
2. Compute the X_{ij} factors for each visibility integration period within the observation.
3. Determine the antenna complex gain factors for each integration period.
4. Use the gains to calibrate the observed visibility values and make an image.
5. Use CLEAN and select components to provide positivity and confinement of the image.
6. Test for convergence and return to step 2 as necessary.

Self-calibration, like basic calibration, requires that the signal-to-noise ratio on each baseline be of the order of at least 3–5 or so. For weak sources, achieving this SNR may necessitate using a long solution interval during which the gains are assumed to remain constant. However, if the gains vary on a timescale significantly shorter than the solution interval, such variations can degrade image quality. In such cases, further improvement in image quality through self-calibration will not be possible.

The self-calibration in my analysis

The self-calibration was performed by using the `gaincal` and `applycal` tasks in the CASA tools. Table 2.11 shows the options set in each iteration. The description of options is available at the CASA homepage ([casatasks 2020](https://casatasks.github.io)). The option `gaintype` was set to ‘T’ in every iteration.

Table 2.11: Self-calibration options

iteration	calmode	solint	combine	refant	applymode
CC 1	‘p’	‘inf’	‘spw’	‘DV05, DV21, DV17’	calflag
CC 2	‘p’	180s	‘spw’	‘DV05, DV21, DV17’	calonly
CC 3	‘p’	180s	‘spw’	‘DV05, DV21, DV17’	calflag
CC 4	‘p’	90s	‘spw’	‘DV05, DV21, DV17’	calflag
CC 5	‘ap’	‘inf’	‘obs, scan’	‘DV05, DV21, DV17’	calflag
concat 1	‘p’	‘inf’	‘obs, scan’	‘DA41’	calflag

Appendix 2.8 : Dust Continuum Maps

Figure 2.26 (a) and (b) show the dust continuum image in the EC data and the CC data, respectively. I summarize properties of each dust continuum image in table 2.12.

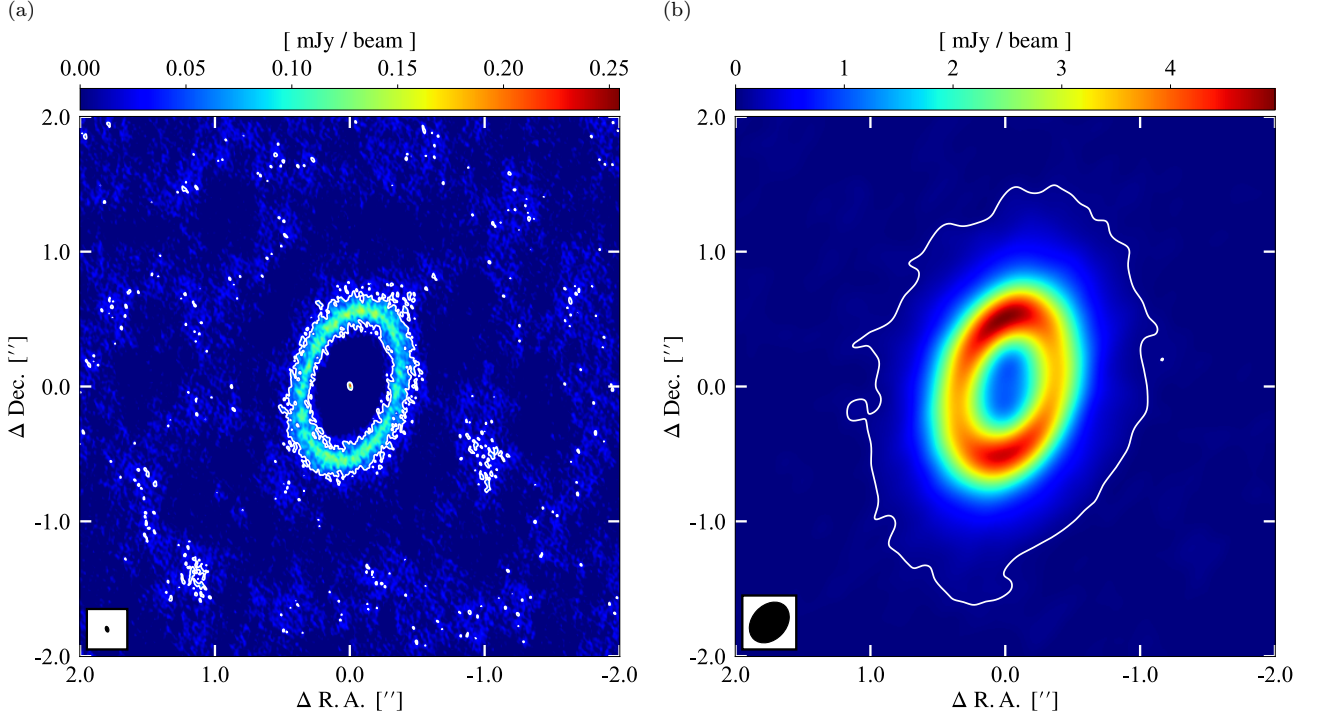


Figure 2.26: Dust continuum emission in (a) the EC data and (b) CC data. The black ellipse at the bottom left of each panel represents the beam size. The white contour denotes the 3σ , where the 1σ level is the rms noise (EC data: $13.6 \mu\text{Jy}/\text{beam}$, CC data: $26.7 \mu\text{Jy}/\text{beam}$).

Table 2.12: Properties of each dust continuum image

	EC data	CC data
Beam size	$0.043'', 0.024''$ 17.8°	$0.337'', 0.254''$ -42.9°
rms [$\mu\text{Jy}/\text{beam}$]	13.6	26.7
Total Flux [mJy]	37.2	56.0

Appendix 2.9 : Visibility analysis of axisymmetric disks

With reference to [Thompson et al. \(2017\)](#), I explain the visibility analysis of azimuthally symmetric disks.

Hankel transform

The Hankel transform of order zero is an integral transform equivalent to a two-dimensional Fourier transform with a radially symmetric integral kernel.

The Fourier transform between $f(x, y)$ and $F(u, v)$ is defined as follows:

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-2\pi i(ux+vy)} dx dy, \quad (2.34)$$

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{2\pi i(ux+vy)} du dv. \quad (2.35)$$

The transform can be converted to polar coordinates by defining $x = r \cos \theta$, $y = r \sin \theta$, $u = q \cos \phi$, and $v = q \sin \phi$, which is written

$$F(q, \phi) = \int_0^{\infty} \int_0^{2\pi} f(r, \theta) e^{-2\pi i r q \cos(\theta-\phi)} r dr d\theta. \quad (2.36)$$

If $f(r, \theta)$ is azimuthally symmetric (i.e. $f(r, \theta) = f(r)$), then

$$F(q, \phi) = \int_0^{\infty} f(r) r dr \int_0^{2\pi} e^{-2\pi i r q \cos(\theta-\phi)} d\theta. \quad (2.37)$$

Since the zeroth-order Bessel function is defined as

$$J_0(z) = \frac{1}{2\pi} \int_0^{2\pi} e^{-iz \cos \theta} d\theta, \quad (2.38)$$

the Hankel transform can be written as

$$F(q) = 2\pi \int_0^{\infty} f(r) J_0(2\pi q r) r dr, \quad (2.39)$$

$$f(r) = 2\pi \int_0^{\infty} F(q) J_0(2\pi q r) q dq. \quad (2.40)$$

Axisymmetric disks

If the disk has azimuthal symmetry (i.e., $I(l, m) = I(r)$, where $r = \sqrt{l^2 + m^2}$), the Fourier transform between the intensity and visibility becomes a Hankel transform as follows:

$$\mathcal{V}_{\text{real}}(q) = 2\pi \int_0^{\infty} I(r) J_0(2\pi r q) r dr, \quad (2.41)$$

$$I(r) = 2\pi \int_0^{\infty} \mathcal{V}_{\text{real}}(q) J_0(2\pi r q) q dq, \quad (2.42)$$

where $q = \sqrt{u^2 + v^2}$. $\mathcal{V}_{\text{real}}(q)$ is a real part of the visibility, and that's imaginary part is zero.

First, the considering a simple model, which is one of a uniform bright circular source of intensity I_0 and radius a , is useful for constructing of a more complicated disk model. Since

$$\int J_0(x) x dx = x J_1(x), \quad (2.43)$$

$$\mathcal{V}_{\text{real}}(q) = \pi a^2 I_0 \frac{J_1(2\pi a q)}{\pi a q}. \quad (2.44)$$

The visibility of an annulus of inner and outer radii a_1 and a_2 can be represented as the difference of two disk visibility functions

$$\mathcal{V}_{\text{real}}(q) = \pi a_2^2 I_0 \frac{J_1(2\pi a_2 q)}{\pi a_2 q} - \pi a_1^2 I_0 \frac{J_1(2\pi a_1 q)}{\pi a_1 q}. \quad (2.45)$$

A useful model for the analysis of an azimuthally symmetric source might be a superposition of annuli in image space with intensities I_i and outer and inner radii of a_i and a_{i-1} . The inner radius of the innermost annuli is zero, so it is a disk. The visibility function is

$$\mathcal{V}_{\text{real}}(q) = \pi \sum_{i=0}^{n-1} (I_i - I_{i+1}) a_i^2 \frac{J_1(2\pi a_i q)}{\pi a_i q} + \pi I_n a_n^2 \frac{J_1(2\pi a_n q)}{\pi a_n q}. \quad (2.46)$$

For the case of a uniform disk, all the I_i s are the same, i.e., I_0 , and the visibility is that of a uniform disk of radius a_n and intensity I_0 ,

$$\mathcal{V}_{\text{real}}(q) = \pi a_n^2 I_0 \frac{J_1(2\pi a_n q)}{\pi a_n q}, \quad (2.47)$$

as expected.

Appendix 2.10 : Model Fitting Method

Bayesian modeling

Let θ represent an unknown parameter vector of interest. In Bayesian modeling, we treat θ as a random variable with an associated prior distribution, $p(\theta)$. This prior distribution reflects our beliefs or assumptions about θ before observing any data.

Now, suppose we have observed data, denoted $\mathbf{x}$, with a likelihood function given by $p(\mathbf{x}|\theta)$. The joint distribution of θ and $\mathbf{x}$ is then expressed as

$$p(\theta, \mathbf{x}) = p(\theta)p(\mathbf{x}|\theta), \quad (2.48)$$

To find the marginal distribution of the data, $\mathbf{x}$, we integrate out θ from the joint distribution:

$$p(\mathbf{x}) = \int_{\theta} p(\theta)p(\mathbf{x}|\theta)d\theta. \quad (2.49)$$

The posterior distribution of θ given the observed data, $p(\theta|\mathbf{x})$, can then be computed using Bayes's Theorem:

$$p(\theta|\mathbf{x}) = \frac{p(\theta)p(\mathbf{x}|\theta)}{p(\mathbf{x})} = \frac{p(\theta)p(\mathbf{x}|\theta)}{\int_{\theta} p(\theta)p(\mathbf{x}|\theta)d\theta}. \quad (2.50)$$

In Bayesian analysis, much of the focus lies in understanding this posterior distribution $p(\theta|\mathbf{x})$, which provides updated information about θ in light of the observed data $\mathbf{x}$. However, calculating the posterior distribution can be challenging. While it may be manageable in low-dimensional cases, the computation becomes intractable in higher dimensions. Specifically, evaluating the marginal distribution $p(\mathbf{x})$ —as required by Bayes's theorem—often proves extremely difficult or even infeasible in practice.

Fortunately, since $p(\mathbf{x})$ serves merely as a normalizing constant independent of θ (given the chosen generative model), it is possible to sample from the posterior distribution $p(\theta|\mathbf{x})$ without explicitly computing $p(\mathbf{x})$. This observation is essential because calculating $p(\mathbf{x})$ is generally computationally prohibitive.

As will be explained in the next section, methods like Markov Chain Monte Carlo (MCMC) enable us to sample from the posterior distribution without needing to know the exact value of the marginal distribution, $p(\mathbf{x})$.

Markov Chain Monte Carlo methods

With reference to [Hogg et al. \(2010\)](#), I explain the MCMC method.

First, the idea of Monte Carlo sampling is the following. The expectation value of a function of the model parameters $f(\theta)$ is

$$\langle f(\theta) \rangle = \int p(\theta|\mathbf{x})f(\theta)d\theta \simeq \frac{1}{M} \sum_i^M f(\theta^i). \quad (2.51)$$

Instead of trying to deal with intractable computations involving the posterior, we can get samples from this distribution (using only the not normalised part definition) and use these samples to compute various punctual statistics such as mean and variance.

MCMC is a procedure for generating a random walk in the parameter space that, over time, draws a representative set of samples from the distribution. Each point in a Markov chain $X(t_i) = \theta^i$ depends only on the position of the previous step $X(t_{i-1})$.

Metropolis Hastings algorithm

The Metropolis Hastings algorithm, a Markov chain Monte Carlo (MCMC) method, is one of the most popular techniques. The algorithm allows us to sample from an unknown distribution if we have known a function that is proportional to the probability density of the distribution we want to sample from.

We give the detailed balance condition for Markov chains written by

$$p(\theta)Q(\theta^*|\theta) = p(\theta^*)Q(\theta|\theta^*), \quad (2.52)$$

where $p(\theta)$ is a target distribution, and $Q(\theta^*|\theta)$ is a transition probability going from θ to θ^* . If $p(\theta)$ satisfies this then $p(\theta)$ is a stationary distribution of the Markov chain. Moreover, using $\pi(\theta)$ to represent a function that is proportional to $p(\theta)$, we obtain

$$\frac{Q(\theta^*|\theta)}{Q(\theta|\theta^*)} = \frac{p(\theta^*)}{p(\theta)} = \frac{\pi(\theta^*)}{\pi(\theta)}. \quad (2.53)$$

Here, we break up $Q(\theta^*|\theta)$ into two independent steps: a proposal distribution $q(\theta^*|\theta)$ and an acceptance distribution $A(\theta^*|\theta)$. Since they are independent, the transition probability is just the multiplication of the two:

$$Q(\theta^*|\theta) = q(\theta^*|\theta)A(\theta^*|\theta). \quad (2.54)$$

Since $q(\theta^*|\theta)$ decides the next point we will potentially be sampling, it's important that it have the same support as the target distribution $p(\theta)$ (ergodicity condition). A typical choice here would be the normal distribution centered on the current state. Now given a fixed proposal distribution $q(\theta^*|\theta)$, we wish to find an $A(\theta^*|\theta)$ that matches. Re-writing Equation (2.53) and substituting in Equation (2.54):

$$\frac{A(\theta^*|\theta)}{A(\theta|\theta^*)} = \frac{\pi(\theta^*)q(\theta|\theta^*)}{\pi(\theta)q(\theta^*|\theta)}. \quad (2.55)$$

Although not obvious, a typical choice for A that satisfies this equation is

$$A(\theta^*|\theta) = \min \left[1, \frac{\pi(\theta^*)q(\theta|\theta^*)}{\pi(\theta)q(\theta^*|\theta)} \right]. \quad (2.56)$$

Because the detailed balance is satisfied by the construction, the steady state of the generated Markov chain will be equal to the target distribution $p(\theta)$. The scheme for the Metropolis Hastings algorithm is presented in Algorithm 3.

Algorithm 3 Metropolis Hastings method

Initialize:

Set the initial parameter θ^0

Repeat:

- Generate uniform random number: $u \sim U(0, 1)$
- Generate a proposed value θ^* from proposal probability $q(\theta^*|\theta^i)$
- Compute an acceptance probability: $A = \min \left[1, \frac{\pi(\theta^*)q(\theta^i|\theta^*)}{\pi(\theta^i)q(\theta^*|\theta^i)} \right]$
- if $u < A$
 $\theta^{i+1} = \theta^*$ (Accept the proposed value)
- otherwise
 $\theta^{i+1} = \theta^i$ (Reject the proposed value)

Until:

Continue until sufficient number of samples $\{\theta^i\}$ are generated

For symmetric q (i.e. $q(\theta^*|\theta) = q(\theta|\theta^*)$), equation (2.56) becomes

$$A(\theta^*|\theta) = \min \left[1, \frac{\pi(\theta^*)q(\theta|\theta^*)}{\pi(\theta)q(\theta^*|\theta)} \right] = \min \left[1, \frac{\pi(\theta^*)}{\pi(\theta)} \right]. \quad (2.57)$$

When the acceptance probability can be written by equation (2.57), the algorithm is called Metropolis method. Figure 2.27 shows the MCMC sampling of two-dimensional parameter using Metropolis algorithm. The target distribution is the two-dimensional Gaussian distribution minus the normalization constants (figure 2.27(a)). After setting the parameters, I reflect transition destinations candidate by rejecting and accepting based on the acceptance probability (figure 2.27(b) and (c)). Figure 2.27(d) shows the samples in step 10000, which are concentrated on center and have a distribution close to the target distribution.

The Metropolis Hastings algorithm converges (as $t \rightarrow \infty$) to a steady distribution of samples from the distribution but there are many algorithms with faster convergence and varying levels of implementation difficulty. Faster convergence is preferred because of the reduction of computational cost due to the smaller number of likelihood computations necessary to obtain the equivalent level of accuracy.

Stretch move algorithm

As mentioned in chapter 3.3, I performed the model fitting of the observed visibility by using `emcee` (Foreman-Mackey et al. 2013). The default move of `emcee` is the “stretch move” ensemble method from Goodman & Weare (2010). This method involves simultaneously evolving an ensemble of K walkers $S = \theta^k$ where the proposal distribution for one walker k is based on the current positions of the $K - 1$ walkers in the complementary ensemble $S_{[k]} = \{\theta^i, \forall i \neq k\}$. Here, position refers to a vector in the N -dimensional, real-valued parameter space.

To update the position of a walker at position θ^k , a walker θ^i is drawn randomly from the remaining walkers $S_{[k]}$ and a new position is proposed:

$$\theta^k(t) \rightarrow \theta^* = \theta^i + Z[\theta^k(t) - \theta^i] \quad (2.58)$$

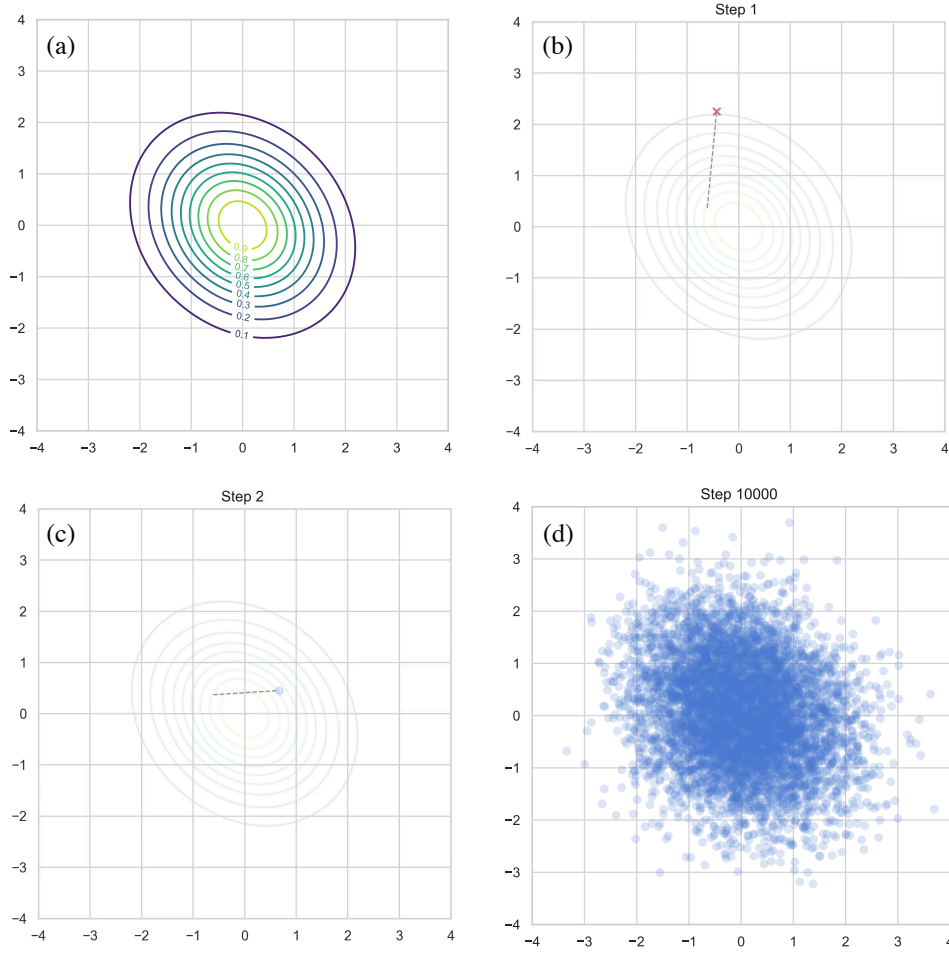


Figure 2.27: Schematic figure of MCMC sampling from the two parameter distribution. The horizontal and vertical axes are the two parameters. (a): Non-normalized target distribution in the shape of the bivariate Gaussian is displayed by contours. (b): The red sample represents a rejected one. The dashed line represent the movement path from the current position to the next candidate position. (c): The blue sample represents an accepted one. (d): MCMC samples in step 10000. The samples are concentrated on center and have a distribution close to the target distribution shown in (a).

where Z is a random variable drawn from a distribution $g(Z = z)$. It is clear that if g satisfies

$$g(z^{-1}) = zg(z), \quad (2.59)$$

the proposal of equation (2.58) is symmetric. In this case, the chain will satisfy detailed balance if the proposal is accepted with probability

$$A = \min \left(1, Z^{N-1} \frac{p(\theta^*)}{p(\theta^k(t))} \right), \quad (2.60)$$

where N is the dimension of the parameter space. This procedure is then repeated for each walker in the ensemble in series following the procedure shown in Algorithm 4.

Goodman & Weare (2010) advocate a particular form of $g(z)$, namely

$$g(z) = \begin{cases} \frac{1}{\sqrt{z}} & \text{if } z \in [\frac{1}{a}, a] \\ 0 & \text{otherwise} \end{cases} \quad (2.61)$$

where a is an adjustable scale parameter that Goodman & Weare (2010) set to 2.

Algorithm 4 Strech move

Initialize:

Set the initial parameter $\theta^k(0)$

Repeat:

- Draw a walker θ^i at random from the complementary ensemble $S_{[k]}(t)$
- Generate uniform random number: $u \sim U(0, 1)$
- Generate a proposed value θ^* from $\theta^i + Z[\theta^k(t) - \theta^i]$
- Compute an acceptance probability: $A = \min\left(1, Z^{N-1} \frac{p(\theta^*)}{p(\theta^k(t))}\right)$
- if $u < A$
 - $\theta^k(t+1) = \theta^*$ (Accept the proposed value)
- otherwise
 - $\theta^k(k+1) = \theta^k(t)$ (Reject the proposed value)

Until:

Continue until sufficient number of samples $\{\theta^k(t)\}$ are generated

Chapter 3

Shadow-Based Framework for Estimating Transition Disk Geometries

This chapter is based on a paper accepted for publication in the Astrophysical Journal (ApJ) and currently available on arXiv: [arXiv:2505.06044] (Orihara & Momose 2025).

ABSTRACT

Some transition disks host misaligned inner disks with radii of several au. Understanding the geometric and physical properties of these misaligned disks is essential for advancing models of terrestrial planet formation. This chapter introduces a novel method to infer the three-dimensional structures of both inner and outer disks by analyzing non-axisymmetric shadows and a horizon in optical and infrared (OIR) scattered light images of the outer disk. The method was applied to the HD 100453 and HD 142527 systems, where infrared scattered light images from the Very Large Telescope (VLT) revealed disk shadows. The results indicate that the inner disk is misaligned by $> 50^\circ$ relative to the outer disk, consistent with previous studies. The aspect ratio of the inner disk was estimated to be > 0.15 , which likely represents the surface height of the optically thick dusty component at the dead zone inner edge. Additionally, the surface height distribution of the outer disk was characterized, providing novel insights into its vertical structure.

3.1 Introduction

Recent observations at radio and optical and infrared (OIR) wavelengths have revealed a wide variety of structures within the protoplanetary disks that surround young stars (Benisty et al. 2023, Pinte et al. 2023). Among these protoplanetary disks, those that have an inner cavity as a result of the dust in the inner region being depleted are called transition disks (Espaillat et al. 2014). Observations of the dust continuum emission with the Atacama Large

Millimeter/submillimeter Array (ALMA) have provided insight into the density and temperature distributions within these disks. The presence of compact dust disks within the cavities of several transition disks has been confirmed. Research by [Francis & van der Marel \(2020\)](#) indicates that about half of the observed sources have an inner disk, suggesting that dust in the innermost regions of disks is either lingering or sustained by a dust supply rate that is comparable to the stellar accretion rate.

However, although compact inner disks have been detected, a limited number of targets have been successfully spatially resolved at a resolution sufficient to reveal their geometry. Attempts have been made to observe the shape of the inner disk directly with Very Large Telescope Interferometer (VLTI) ([Bohn et al. 2022](#)), but these are based on parametric models of visibilities that are required for additional components other than the inner disk, making it difficult to discuss uncertainties on the derived disk parameters. In addition, the use of an interferometer combining four telescopes results in a smaller data sample in the visibility plane compared to ALMA. Therefore, the analysis of inner disk structure based on imaging remains challenge with current instrumentation.

The shadow cast on the outer disk can also have a significant effect on its evolution. According to [Montesinos et al. \(2016\)](#), local cooling caused by the shadow cast by the inner disk can potentially trigger the formation of spiral arms in the outer disk. The localized temperature drop induces an azimuthal pressure gradient. As the gas rotates and moves through the shadowed region, it undergoes azimuthal acceleration, leading to the development of spiral arms. This process can occur even in gravitationally stable disks without planets, indicating the potential for planetesimal formation across a wide range of evolutionary stages.

The framework linking the shadow to the orientation of the inner disk has already been proposed by [Min et al. \(2017\)](#). In this analytical model, the two shadows on the outer disk is calculated under the given position angle (PA) and inclination (i) of both the inner and outer disks and the height of the scattering surface (h) of the outer disk. However, there are several limitations to this method for estimating the shape of the inner disk from its shadow. One such limitation is the neglect of the thickness of the inner disk, which would result in infinitesimally narrow shadows on the disk. Because the actual shadow has a finite width, their model accepts a wide range of inclination angles, resulting in a degeneracy of parameters. Another limitation is the assumption of a constant h/r for the height of the scattering surface of the outer disk, which does not take into account the radial variation that must be present in a real disk. Such variation can significantly change the position of the shadows, and thus the estimation of the shape of the inner disk.

To address these issues, I develop a new methodology that differs from previous studies in two key aspects. First, it incorporates a finite thickness for the inner disk. Second, it accounts for the scattering surface height of the outer disk as a function of radius. In addition, a significant feature of this method is the fitting of the boundary between illuminated and shadowed regions on the outer disk using a curve. This allows for the construction of a more

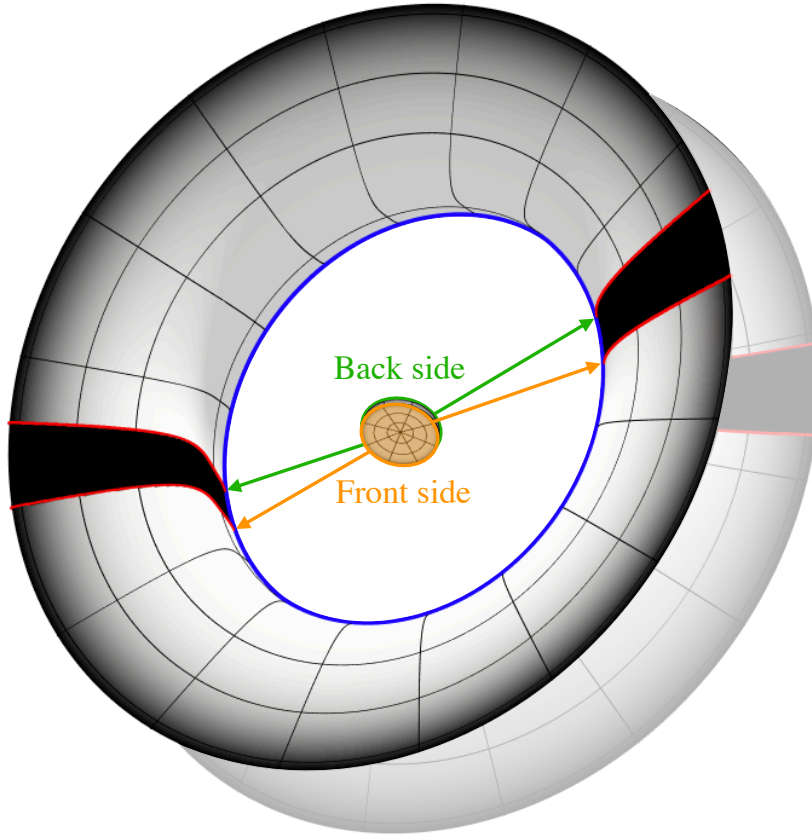


Figure 3.1: Schematic illustration of the scattering surface of the transition disk with the misaligned inner disk. The black regions represent shadows cast by the misaligned inner disk, and the red curves indicate the boundary between the illuminated and shadowed areas. The blue curve represents the horizon curve of the outer disk.

detailed model that incorporates both the position and width of the shadows. The core concept of this method is the horizon of the outer disk surface. The horizon defines the boundary that separates the visible and non-visible regions of the outer disk surface from the observer’s point of view, thereby improving the accuracy of determining the outer disk geometry.

The remaining part of this chapter is organized as follows. The detailed formulation of my method is presented in Section 3.2. The results of the application of this method to the infrared scattered light observations of HD100453 is presented in Section 3.3. In Section 3.4, the results of applying this method to HD 142527 are presented. The physical quantities and implications of the geometric structures derived from my findings are then discussed (Section 3.5). Finally, I summarize and present my conclusions (Section 3.6).

3.2 Methods

This section presents the parameter search methods for exploring the geometric structure of the inner and outer disk surfaces. It provides precise definitions for the horizon and the shadow boundaries, followed by a detailed explanation of the optimization techniques employed to refine the disk geometry model based on these curves.

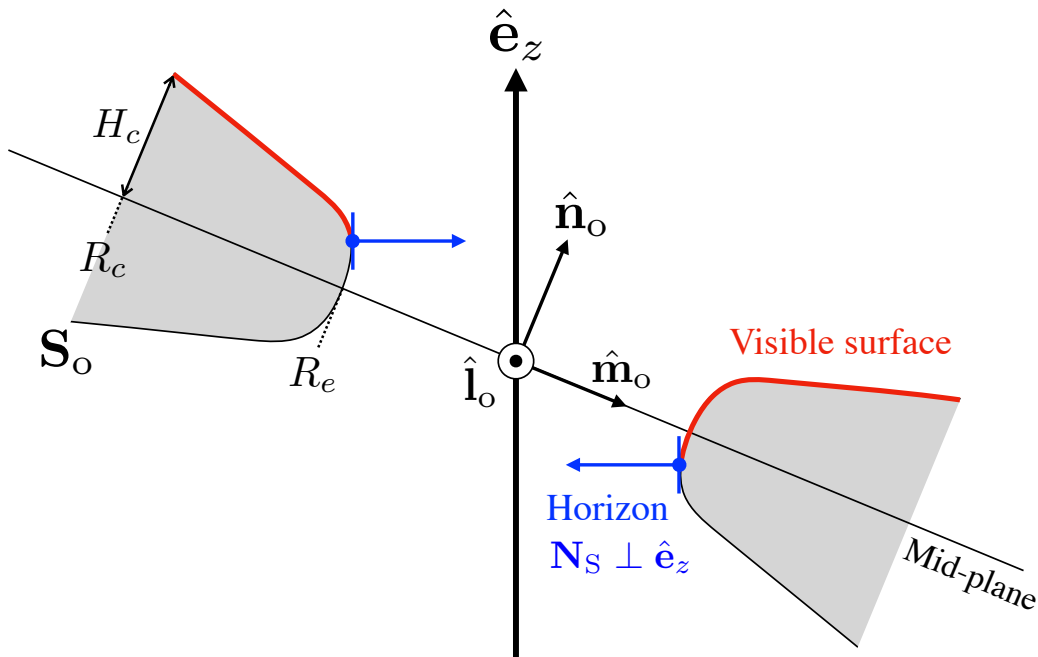


Figure 3.2: Horizon in the optically thick surface of the outer disk cross-section viewed along the major axis ($\hat{\mathbf{l}}_0$ -axis). The surface of the outer disk is modeled such that the radius of the inner edge is R_e , and the surface height at a radius of R_c is H_c . The position of the horizon, marked by the blue dots, is defined by the condition $\mathbf{N}_S \cdot \hat{\mathbf{e}}_z = 0$. The red curve represents the visible surface, and the surface beyond the horizon is not visible to us.

3.2.1 Horizon of the Outer Disk Surface

The coordinate system is defined such that the positive x-axis points towards the North, the positive y-axis towards the East, and the positive z-axis towards the observer. In this system, the unit vector $\hat{\mathbf{e}}_z$ represents the line of sight, pointing from the celestial sphere towards the observer, while the vectors $\hat{\mathbf{e}}_x$ and $\hat{\mathbf{e}}_y$ correspond to the directions of declination (Dec) and right ascension (RA), respectively.

The normal vector of the mid-plane of the outer disk is aligned with the direction

$$\hat{\mathbf{n}}_o = \begin{bmatrix} -\sin(i_o) \sin(\text{PA}_o) \\ \sin(i_o) \cos(\text{PA}_o) \\ \cos(i_o) \end{bmatrix}, \quad (3.1)$$

where i_o is the inclination of the outer disk and PA_o is the position angle of its major axis (Figure 3.2). In addition to this $\hat{\mathbf{n}}_o$, I define $\hat{\mathbf{l}}_o$ and $\hat{\mathbf{m}}_o$ as follows as a basis for describing disk geometry:

$$\hat{\mathbf{l}}_o = \frac{\hat{\mathbf{n}}_o \times \hat{\mathbf{e}}_z}{|\hat{\mathbf{n}}_o \times \hat{\mathbf{e}}_z|}, \quad (3.2)$$

$$\hat{\mathbf{m}}_o = \hat{\mathbf{n}}_o \times \hat{\mathbf{l}}_o. \quad (3.3)$$

Here, $\hat{\mathbf{l}}_0$ represents the direction parallel to the major axis of the outer disk.

I assume that the outer disk is optically thick at the wavelength where the scattered light is observed. With these basis vectors $\{\hat{\mathbf{l}}_o, \hat{\mathbf{m}}_o, \hat{\mathbf{n}}_o\}$, the surface of the outer disk $\mathbf{S}_o$ in Figure 3.2, centered at $\mathbf{c}_o$, can be described as a function of vertical height H and azimuthal angle ϕ_o :

$$\mathbf{S}_o = R(H)(\cos(\phi_o)\hat{\mathbf{l}}_o + \sin(\phi_o)\hat{\mathbf{m}}_o) + H\hat{\mathbf{n}}_o + \mathbf{c}_o. \quad (3.4)$$

The position $\mathbf{c}_o = (c_{ox}, c_{oy}, c_{oz})$ represents the center of the outer disk relative to the image center. Based on the offsets c_{ox} and c_{oy} on the celestial sphere, the coordinate c_{oz} can be calculated using the following equation:

$$c_{oz} = -\frac{\hat{\mathbf{n}}_o \cdot (c_{ox}\hat{\mathbf{e}}_x + c_{oy}\hat{\mathbf{e}}_y)}{\hat{\mathbf{n}}_o \cdot \hat{\mathbf{e}}_z}. \quad (3.5)$$

This equation is derived under the assumption that the central star lies on the mid-plane of the outer disk.

R represents the cylindrical radius associated with the height H :

$$R(H) = \left[R_e^\alpha + (R_c^\alpha - R_e^\alpha) \left(\frac{|H|}{H_c} \right)^\alpha \right]^{\frac{1}{\alpha}}, \quad (3.6)$$

where R_e denotes the radius of the inner edge of the outer disk, R_c is the characteristic radius, H_c is the surface height at R_c , and α is the power-law index that controls the curvature of the surface, as shown in Figure 3.2. The rationale behind the selection of this equation is provided in Appendix 3.1. To express the curvature of the disk, the derivative of $R(H)$ with respect to H is required. This can be expressed as follows:

$$\frac{dR}{dH} = \text{sign}(H) \frac{R_c^\alpha - R_e^\alpha}{H_c^\alpha} R^{1-\alpha} |H|^{\alpha-1}. \quad (3.7)$$

The surface orientation is characterized by the normal vector $\mathbf{N}_S$, which is obtained by taking the cross product of the partial derivatives of $\mathbf{S}_o$ with respect to the surface height H and the azimuthal angle ϕ_o :

$$\mathbf{N}_S = \frac{\partial \mathbf{S}_o}{\partial H} \times \frac{\partial \mathbf{S}_o}{\partial \phi_o}, \quad (3.8)$$

where

$$\frac{\partial \mathbf{S}_o}{\partial H} = \frac{\partial R}{\partial H} (\cos(\phi_o)\hat{\mathbf{l}}_o + \sin(\phi_o)\hat{\mathbf{m}}_o) + \hat{\mathbf{n}}_o, \quad (3.9)$$

$$\frac{\partial \mathbf{S}_o}{\partial \phi_o} = R(-\sin(\phi_o)\hat{\mathbf{l}}_o + \cos(\phi_o)\hat{\mathbf{m}}_o). \quad (3.10)$$

Using the expressions for the partial derivatives, Equation (3.8) can be rewritten in the following form:

$$\mathbf{N}_S = R \left(-\cos(\phi_o)\hat{\mathbf{l}}_o - \sin(\phi_o)\hat{\mathbf{m}}_o + \frac{\partial R}{\partial H} \hat{\mathbf{n}}_o \right). \quad (3.11)$$

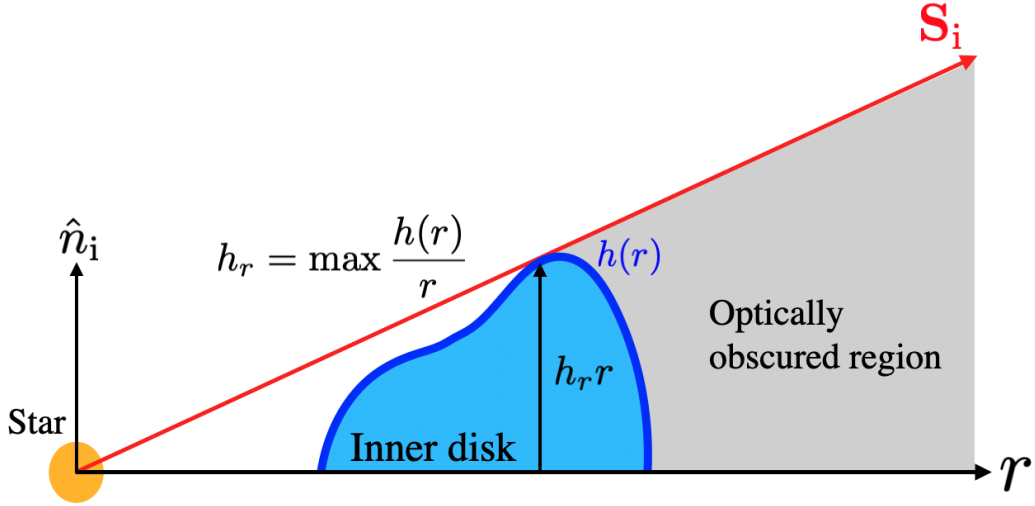


Figure 3.3: Optically obscured region made by the inner disk.

The horizon is the boundary between the visible and non-visible sides of the outer disk's surface, as seen by the observer in scattered light at OIR wavelengths, as shown in Figure 3.2. The horizon of the surface must satisfy the condition that the normal vector of the surface is perpendicular to the line of sight. Therefore, the equation for the horizon is given as follows:

$$\mathbf{N}_S \cdot \hat{\mathbf{e}}_z = 0. \quad (3.12)$$

Recalling that $\hat{\mathbf{l}}_o$ and $\hat{\mathbf{e}}_z$ are orthogonal, this condition can be simplified as the following equation:

$$\begin{aligned} \phi_{o, \text{Hor}} &= \arcsin \left(\frac{\partial R}{\partial H} \frac{\hat{\mathbf{n}}_o \cdot \hat{\mathbf{e}}_z}{\hat{\mathbf{m}}_o \cdot \hat{\mathbf{e}}_z} \right) \\ \text{or } \pi - \arcsin \left(\frac{\partial R}{\partial H} \frac{\hat{\mathbf{n}}_o \cdot \hat{\mathbf{e}}_z}{\hat{\mathbf{m}}_o \cdot \hat{\mathbf{e}}_z} \right) \end{aligned} \quad (3.13)$$

This relation enables the horizon curve on the disk surface, $\mathbf{C}_{\text{Hor}} = \mathbf{S}_o(H; \phi_{o, \text{Hor}}(H))$, to be analytically derived as a function of H . Using this model significantly tightens the constraints on key parameters, such as the central position and the radius of the inner edge of the outer disk.

3.2.2 Shadow Boundary between the Inner and Outer Disks

To capture the three-dimensional shape of the shadow cast by the misaligned inner disk, I model the boundary between the illuminated and shadowed regions on the outer disk. As shown in Figure 3.3, the shadow boundary is mathematically defined as the intersection between the surface of the outer disk and the surface of the optically obscured region made by the inner disk. As a result, the shadow projected onto the outer disk does not capture the optically thick surface profile of the inner disk $h(r)$, but it does provide information about the maximum aspect ratio $h_r = \max h(r)/r$.

Before formulating the disk model, I redefine the basis vectors for each disk. The normal vector of the inner disk, $\hat{\mathbf{n}}_i$, is defined by the inclination i_i and the position angle PA_i , as given in Equation (3.1). These vectors are crucial for computing the misalignment angle between the disks, a key parameter in my model. The misalignment angle, $\Delta\phi_{\text{mis}}$, is calculated using the dot product of $\hat{\mathbf{n}}_o$ and $\hat{\mathbf{n}}_i$, as given by the following equation:

$$\begin{aligned}\Delta\phi_{\text{mis}} &= \arccos[\hat{\mathbf{n}}_o \cdot \hat{\mathbf{n}}_i] \\ &= \arccos[\sin(i_i) \sin(i_o) \cos(\text{PA}_i - \text{PA}_o) \\ &\quad + \cos(i_i) \cos(i_o)].\end{aligned}\tag{3.14}$$

Given $\hat{\mathbf{n}}_o$ and $\hat{\mathbf{n}}_i$, the remaining vectors $\hat{\mathbf{l}}_o$, $\hat{\mathbf{m}}_o$, $\hat{\mathbf{l}}_i$, and $\hat{\mathbf{m}}_i$ are defined as follows:

$$\hat{\mathbf{l}}_i = \hat{\mathbf{l}}_o = \frac{\hat{\mathbf{n}}_o \times \hat{\mathbf{n}}_i}{|\hat{\mathbf{n}}_o \times \hat{\mathbf{n}}_i|},\tag{3.15}$$

$$\hat{\mathbf{m}}_i = \hat{\mathbf{n}}_i \times \hat{\mathbf{l}}_i,\tag{3.16}$$

$$\hat{\mathbf{m}}_o = \hat{\mathbf{n}}_o \times \hat{\mathbf{l}}_o.\tag{3.17}$$

To derive the shadow intersection curve $\mathbf{C}_{\text{Sha}}$, the model accounts for the height profiles of both the inner and outer disks. Using the defined basis vectors $\{\hat{\mathbf{l}}_i, \hat{\mathbf{m}}_i, \hat{\mathbf{n}}_i\}$, the surface $\mathbf{S}_i$ of the optically obscured region caused by the inner disk can be expressed as follows:

$$\mathbf{S}_i = r(\cos(\phi_i)\hat{\mathbf{l}}_i + \sin(\phi_i)\hat{\mathbf{m}}_i) \pm h_r r \hat{\mathbf{n}}_i + \mathbf{c}_i,\tag{3.18}$$

where r , ϕ_i , and $\mathbf{c}_i$ denote the cylindrical radius, azimuthal angle in the mid-plane of the inner disk, and the position vector of the inner disk center, respectively. For simplicity, it is assumed that the center of the inner disk coincides with the position of the central star. Similarly to $\mathbf{c}_o$ on the outer disk, given the offsets c_{ix} and c_{iy} on the celestial sphere, c_{iz} can be computed as follows:

$$c_{iz} = -\frac{\hat{\mathbf{n}}_o \cdot (c_{ix}\hat{\mathbf{e}}_x + c_{iy}\hat{\mathbf{e}}_y)}{\hat{\mathbf{n}}_o \cdot \hat{\mathbf{e}}_z},\tag{3.19}$$

where I assume that a star is located at the center of the inner disk and lies on the mid-plane of the outer disk, leading to the relation $(\mathbf{c}_o - \mathbf{c}_i) \cdot \hat{\mathbf{n}}_o = 0$. Furthermore, since the system is defined by $\mathbf{c}_o \cdot \hat{\mathbf{n}}_o = 0$ throughout, the relation $\mathbf{c}_i \cdot \hat{\mathbf{n}}_o = 0$ must also hold. The $\pm$ sign in the third term of Equation (3.18) denotes the front and back surfaces of the optically obscured region relative to our line-of-sight.

The condition for the intersection curve between the surfaces of the outer disk and the optically obscured region made by the inner disk is given as follows:

$$\mathbf{S}_o - \mathbf{S}_i = 0.\tag{3.20}$$

By calculating this equation as shown in Appendix 3.2, the shadow boundary curves, $\mathbf{C}_{\text{Sha}} = \mathbf{S}_o(H; \phi_{o,\text{Sha}}(H))$, can be derived analytically.

Table 3.1: Model Parameters

Parameter	Description	Unit
c_{ox}	Offset of the outer disk center in the x-direction (Dec)	[au]
c_{oy}	Offset of the outer disk center in the y-direction (RA)	[au]
R_e	Radius of the inner edge of the outer disk	[au]
H_c	Height of the outer disk surface at R_c	[au]
α	Power index of the outer disk surface	[]
c_{ix}	Offset of the inner disk center in the x-direction (Dec)	[au]
c_{iy}	Offset of the inner disk center in the y-direction (RA)	[au]
i_i	Inclination of the inner disk	[°]
PA_i	Position angle of the inner disk	[°]
h_r	Maximum aspect ratio of the inner disk	[]

3.2.3 Gradients of Infrared Scattered Light Image

To estimate the geometric structure of the disk from the observational image, it is necessary to fit curves to the image. In this analysis, the fitting is not performed on the brightness values themselves, but rather on their gradients, as features such as the horizon and the shadow boundaries appear as the regions with the steepest brightness changes in the image. Therefore, it is essential to first extract the pixels with high gradient values from the image.

The process of boundary extraction in OIR scattered light images is often challenging due to the reduced contrast caused by the propagation characteristics of light within scattering media. To address this issue, the present study employs the Contrast Limited Adaptive Histogram Equalization (CLAHE) algorithm (Sasi & Jayasree 2013). CLAHE is effective for enhancing local contrast while mitigating excessive noise amplification. By equalizing the overall contrast of the image, the algorithm is expected to significantly enhance the accuracy of boundary extraction.

To detect the edges of the scattered light intensity I_ν , the Canny method was employed to compute gradient vectors (Canny 1986). The Canny method is a widely used and well-established edge detection technique in the field of image analysis. It consists of several sequential steps, including Gaussian filtering for noise reduction, gradient calculation, non-maximum suppression, and hysteresis thresholding for binarization. However, in this study, only the gradient calculation and non-maximum suppression steps were utilized.

Positions at high gradients were selected, and the directional information of the gradients was analyzed further to separate horizon and shadow components. Specifically, the computed gradient vectors were projected onto the basis vectors $\hat{e}_r$ and $\hat{e}_\phi$ using inner product calculations. Consequently, the $\hat{e}_r$ component of the gradient vector corresponds to the horizon, while the $\hat{e}_\phi$ component corresponds to the shadow's edge.

However, it is important to note that the computation of gradients may also capture small-scale structures other than shadows, such as spirals. To address this potential ambiguity, a mask is applied in advance to constrain the shadow within the outer disk.

3.2.4 Parameter Exploration

I generate the horizon and shadow boundaries curves to find the parameters that minimize the distance between these curves and fast gradient points. The parameters obtained from this modeling process are listed in Table 3.1. For the outer disk, the parameters include c_{ox} , c_{oy} , R_e , H_e , and α . For the inner disk, they include c_{ox} , c_{oy} , i_i , PA_i , and h_r . The inclination and position angle of the outer disk are determined based on the dust continuum emission and CO channel maps observed by ALMA.

To efficiently explore the parameter space and estimate uncertainties, I employ the Markov Chain Monte Carlo (MCMC) method (Foreman-Mackey et al. 2013). To enhance both the efficiency and diversity of the MCMC sampling, I utilize Parallel Tempering. In Parallel Tempering, multiple chains with different “temperature” are run simultaneously, and samples are periodically exchanged between these chains. This method allows the chains to escape from local minima, facilitating broader exploration of the parameter space and enabling the identification of global minima, which are often challenging to capture with conventional MCMC methods.

For the prior distributions of each parameter, I adopt uninformative priors. This approach respects the information derived from the observational data by minimizing the influence of any prior assumptions. The specific prior settings for each parameter are provided in Table 3.2 and 3.3.

3.3 Application to Real Data: HD100453

HD 100453, located at a distance of 103.78 pc (Gaia Collaboration et al. 2021), is a well-studied Herbig Ae star known for its complex disk structure and striking scattered light features. Near-infrared observations have revealed a prominent two-armed spiral structure in the outer disk, as well as asymmetric illumination patterns (Benisty et al. 2017). In addition, the presence of a low-mass stellar companion at a projected separation of ~ 120 au has been proposed as a possible driver of the observed spiral morphology (Rosotti et al. 2020). The disk also exhibits shadows in scattered light, suggesting a misaligned inner disk component (Benisty et al. 2017, Min et al. 2017), making HD 100453 a compelling target for investigating the interaction between inner and outer disk geometries.

By adopting the model presented in Section 3.2, the disks around HD 100453 are modeled from the OIR scattered light image. The image was observed using the high-contrast imaging instrument SPHERE (Spectro-Polarimetric High-contrast Exoplanet REsearch) mounted on

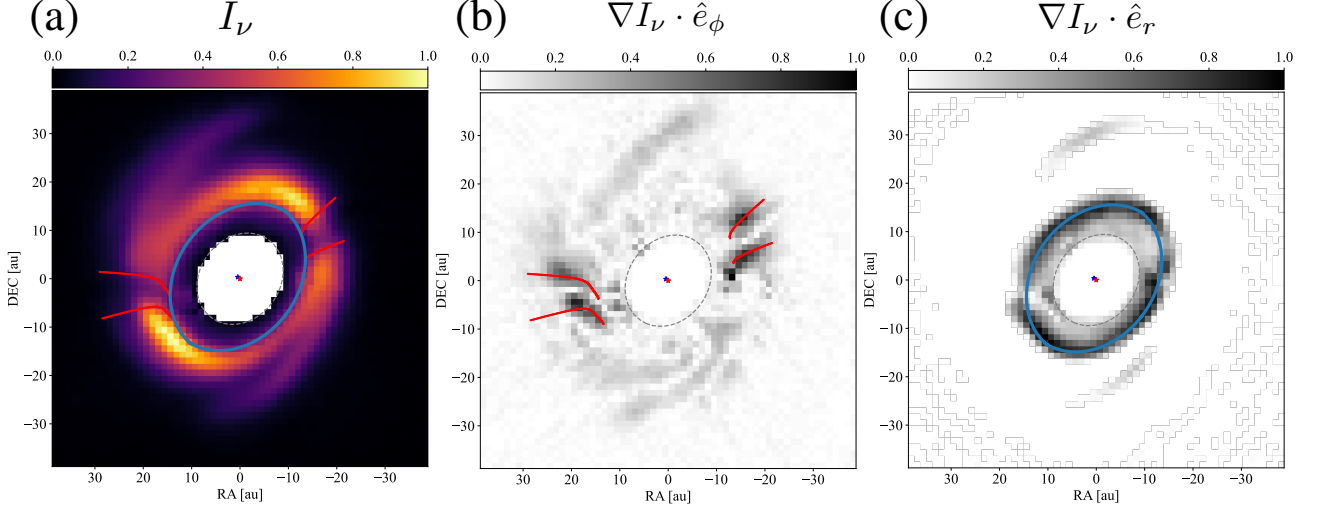


Figure 3.4: The (a), (b), and (c) panels show the infrared scattered light image, the radial gradient image, and the azimuthal gradient image, respectively. All images are displayed in arbitrary units. The blue and red curves represent the optimized horizon curve and shadow intersection curve, respectively. The blue and red star markers at the center indicate the positions of the outer and inner disk, respectively.

the VLT (Very Large Telescope) in the J band under program ID 096.C-0248(B), and was published by [Benisty et al. \(2017\)](#).

The observations were made in polarized intensity. If there was significant variation in the degree of polarization across the image, we should be careful when determining shadow positions. This is especially important for disks with a large misalignment. In the present case, however, the presence of the shadows is too extreme to be explained solely by a decrease in the degree of polarization. I therefore assume that the effect of polarization degree is negligible in the following.

The position of the star (center of the inner disk) is determined with high accuracy using satellite spots obtained through the waffle mode of VLT/SPHERE. During the data reduction process, the star is aligned to the center of the image, so I fix both c_{ix} and c_{iy} to 0. In addition, the PA and i of the outer disk were fixed to 324° and 34° , respectively, based on [Bohn et al. \(2022\)](#).

Figure 3.12 in Appendix 3.3 shows the posterior distributions obtained from the MCMC fitting. The parameters with the maximum posterior probability derived from these distributions are listed in Table 3.2. Figure 3.4 shows the optimized curves generated by my method. In the panel (b), the shadow curve is overlaid on the azimuthal gradient distribution, while in the panel (c), the horizon is overlaid on the radial gradient distribution. These curves are confirmed to trace these distributions of intensity gradients.

In the following, I describe in detail the geometric structure of HD 100453 derived from optimization.

Table 3.2: Model Parameters for the case of HD100453

Parameter	Search range	Solution	Error	Unit
c_{ox}	$[-2, 2]$	0.34	0.016	[au]
c_{oy}	$[-2, 2]$	0.46	0.022	[au]
R_e	$[15, 17.5]$	16.8	0.03	[au]
H_c at $R_c = 25$ au	$[1, 10]$	6.6	0.15	[au]
α	$[2, 5]$	4.0	0.17	[]
c_{ix}	0 (fixed)	-	-	[au]
c_{iy}	0 (fixed)	-	-	[au]
i_i	$[0, 90]$	50.5	0.8	[°]
PA _i	$[0, 180]$	88.3	0.5	[°]
h_r	$[0.05, 0.2]$	0.17	0.001	[]

3.3.1 Inclination and Position Angle of the Inner Disk

The position angle (PA) and inclination (i) of the inner disk were estimated to be 88° and 50° , respectively. These results are generally consistent with measurements of the inclination of the inner disk obtained by VLTI/GRAVITY (Bohn et al. 2022), demonstrating the robustness of this orientation. Furthermore, the misalignment angle of the inner disk relative to the outer disk, as described in Equation (3.15), was estimated to be $\sim 70^\circ$. According to Nealon et al. (2020a), in order to generate such a large tilt, an additional planetary companion of $\sim 5M_J$ is required inside the outer disk, along with the known stellar companion.

3.3.2 Maximum Aspect Ratio of the Inner Disk

The maximum aspect ratio h_r of the inner disk was estimated to be 0.17. Assuming that the star with a luminosity of $6L_\odot$ (Bohn et al. 2022) hosts an optically thin inner disk, the gas pressure scale height at a radius of 1 au is estimated to be approximately 0.03 au. The maximum aspect ratio h_r of the inner disk can be more than five times higher than the gas pressure scale height. This suggests that the number density of small dust particles above the pressure scale height is so high that the dust scattering surface is located higher than the pressure scale height. It is possible that the dust tightly coupled to the gas is vertically dispersed by the gas turbulence or disk wind.

The observed large aspect ratio may originate from structural changes caused by dust accumulation at the dead-zone inner edge. Indeed, Ueda et al. (2019) demonstrated that dust accumulation near the dead-zone inner edge can lead to a maximum in the aspect ratio of the optically thick dust surface.

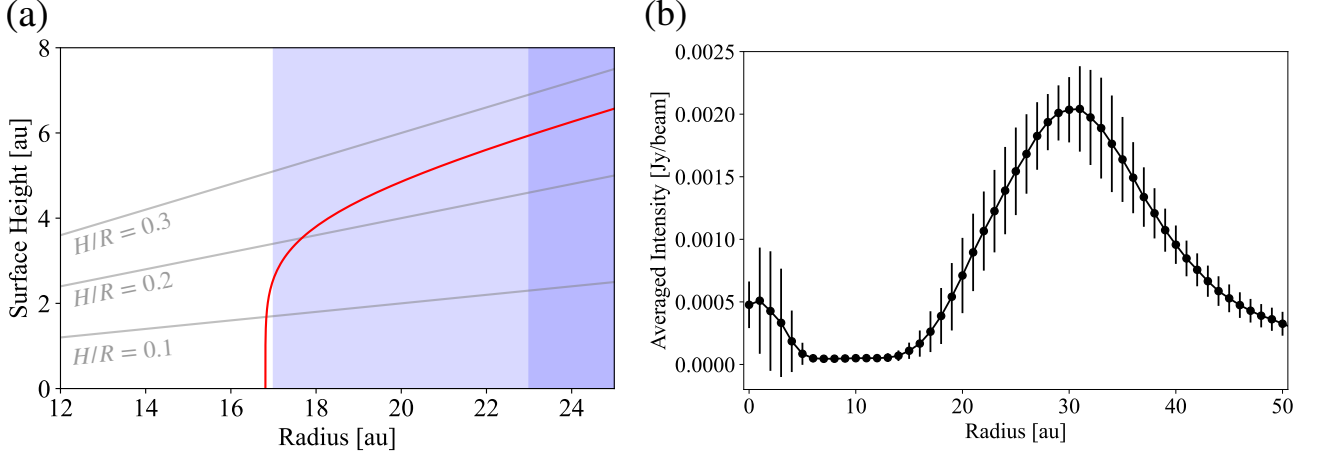


Figure 3.5: (a): Estimated disk surface height profile of HD100453 represented by Equation (3.6) (red line). (b): The azimuthally averaged intensity profile of the dust continuum emission observed with ALMA Band 7 (Rosotti et al. 2020). The error bars represent the standard deviation of the intensity at each radius. The dark blue shaded region indicates areas with an intensity greater than half of the peak intensity, while the light blue shaded region corresponds to intensities greater than one-tenth of the peak intensity. The shaded regions in panel (a) follow the same definitions.

3.3.3 Surface Height Profile of the Outer Disk

The location of the scattering surface of the outer disk is presented in Figure 3.5, and it is evident that the aspect ratio of the outer disk ranges from 0.15 to 0.25. The parameter α was estimated to be ~ 4 , and the height profile converges rapidly to the form $H \propto R$. Additionally, as shown in Figure 3.12, H_c and α exhibit a negative correlation. When H_c is low, α increases to maintain the surface at a relatively high position. Conversely, when H_c is high, α decreases but still keeps the surface at a similar height. This interplay between H_c and α strongly supports the presence of an optically thick dusty surface at high positions near the inner edge of the outer disk.

I compare the distribution of small dust grains traced by scattered light with that of larger dust grains traced by millimeter emission. As illustrated in Figure 3.5 (b), the azimuthally averaged intensity of the dust continuum emission observed in ALMA Band 7 (Rosotti et al. 2020) exhibits an increase around 17 au and a peak near 30 au. The 17 au corresponds to the radius of the inner edge of the outer disk identified in OIR scattered light, whereas the high density of millimeter-sized dust is observed further out, around 30 au. These comparisons suggest that larger dust grains are more likely to be trapped at pressure bumps, while smaller dust grains drift inward to regions closer than this location.

3.4 Application to Real Data: HD142527

HD 142527 is a Herbig Fe star known for its large inner cavity and a highly asymmetric crescent-shaped structure in the outer disk (Fukagawa et al. 2006, Soon et al. 2019). This crescent, clearly observed in submillimeter wavelengths, suggests a strong concentration of

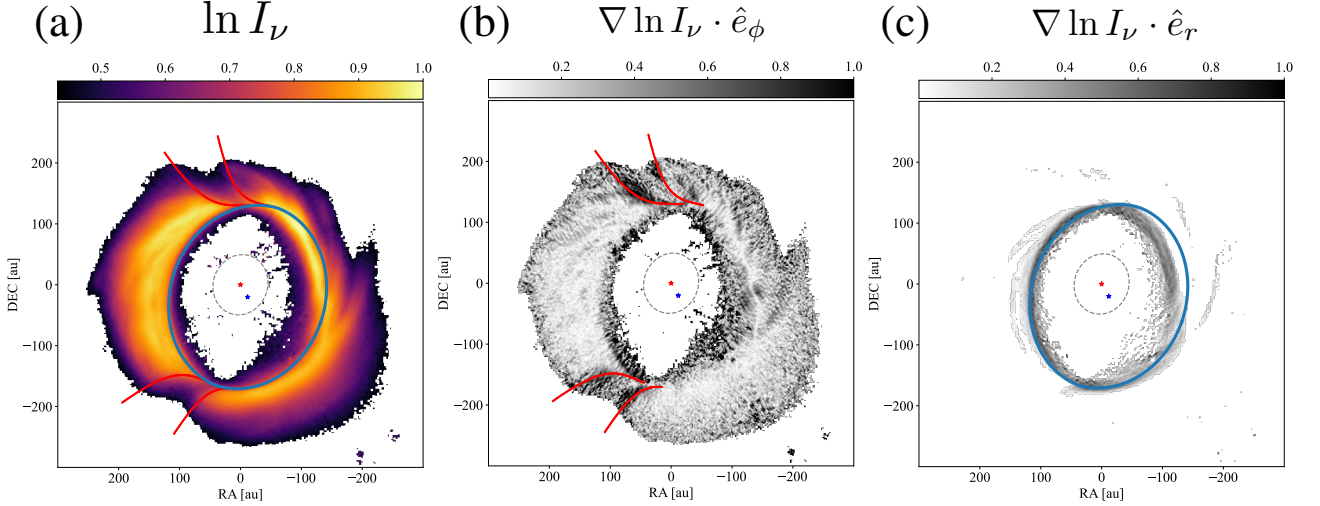


Figure 3.6: The (a), (b), and (c) panels show the infrared scattered light image, the radial gradient image, and the azimuthal gradient image, respectively. All images are displayed in arbitrary units. The blue and red curves represent the optimized horizon curve and shadow intersection curve, respectively. The blue and red star markers at the center indicate the positions of the outer and inner disk, respectively.

dust and the possible presence of a dust trap. In addition, deep shadows have been detected in optical and near-infrared scattered light images, which are thought to originate from a misaligned inner disk (Avenhaus et al. 2017, Casassus et al. 2018, Marino et al. 2015). These complex structural features make HD 142527 a particularly valuable target for studying the geometry of inner and outer disks and their observational manifestations.

By adopting the model presented in Section 3.2, the disks around HD 142527, located at a distance of 159.26 pc (Gaia Collaboration et al. 2021), are modeled from the OIR scattered light image. The image was observed using the high-contrast imaging instrument SPHERE mounted on the VLT in the H band under the SPHERE GTO campaign (Avenhaus et al. 2017), and was published by Hunziker et al. (2021).

Similar to the case of HD 100453, the center of the inner disk is fixed at (0,0), and the orientation of the outer disk is set to $PA = 341^\circ$ and $i = 27^\circ$, following Bohn et al. (2022).

Unlike the case of HD100453, this analysis utilized the logarithmic gradient of intensity for edge detection as shown in Figure 3.6. This approach was adopted because taking the logarithmic derivative normalizes the gradient by intensity, thereby eliminating dependence on absolute brightness, while also enhancing the emphasis on weak shadow boundaries.

The disk surrounding HD142527 exhibits distinct non-axisymmetric features resembling spiral structures, which cause the horizon to deviate from an elliptical shape. Therefore, I utilized only the eastern side of the disk, which is minimally affected by the spirals, for the horizon-fitting process.

Figure 3.13 in Appendix 3.3 shows the posterior distributions obtained from the MCMC fitting. The parameters with the maximum posterior probability derived from these distributions are listed in Table 3.3. Figure 3.6 shows the optimized curves generated by my method. In the panel (b), the shadow curve is overlaid on the azimuthal gradient distribution, while in

the panel (c), the horizon is overlaid on the radial gradient distribution.

In the following, I describe in detail the geometric structure of HD 142527 derived from optimization.

Table 3.3: Model Parameters for the case of HD142527

Parameter	Search range	Solution	Error	Unit
c_{ox}	$[-30, 0]$	-20.2	0.05	[au]
c_{oy}	$[-30, 0]$	-11.9	0.13	[au]
R_e	$[140, 160]$	154	0.08	[au]
H_c at $R_c = 250$ au	$[10, 150]$	107	0.6	[au]
α	$[2, 5]$	2.87	0.014	[]
c_{ix}	0 (fixed)	—	—	[au]
c_{iy}	0 (fixed)	—	—	[au]
i_i	$[0, 90]$	28.9	0.2	[°]
PA _i	$[0, 300]$	180.2	0.1	[°]
h_r	$[0.01, 0.2]$	0.15	0.001	[]

3.4.1 Geometric Structure of the Inner Disk

The position angle (PA) and inclination (i) of the inner disk were estimated to be 180° and 29° , respectively. These results are generally consistent with measurements of the inclination of the inner disk obtained by VLTI/GRAVITY (Bohn et al. 2022), demonstrating the robustness of this orientation. Furthermore, the misalignment angle of the inner disk relative to the outer disk, as described in Equation (3.15), was estimated to be $\sim 55^\circ$.

The maximum aspect ratio of the inner disk was estimated to be 0.15, which corresponds to twice the gas scale height. This suggests that the amount of small dust present above the gas scale height is so significant that the dust scattering surface is located higher. It is possible that dust coupled with the gas is being vertically dispersed by the gas turbulence or the disk wind.

3.4.2 Surface Height Profile of the Outer Disk

The location of the scattering surface of the outer disk is shown in Figure 3.7, clearly indicating that the aspect ratio of the outer disk ranges from 0.15 to 0.4. The parameter α was estimated to be approximately 3, and the height profile rapidly converges to the form $H \propto R$.

As illustrated in Figure 3.7 (b), the azimuthally averaged intensity of the dust continuum emission observed in ALMA Band 9 (Casassus et al. 2015) exhibits an increase around 120 au and a peak near 170 au. The radial intensity peak is located outside the inner edge radius R_e of 150 au identified in OIR scattered light, but weak emission is also observed inside this

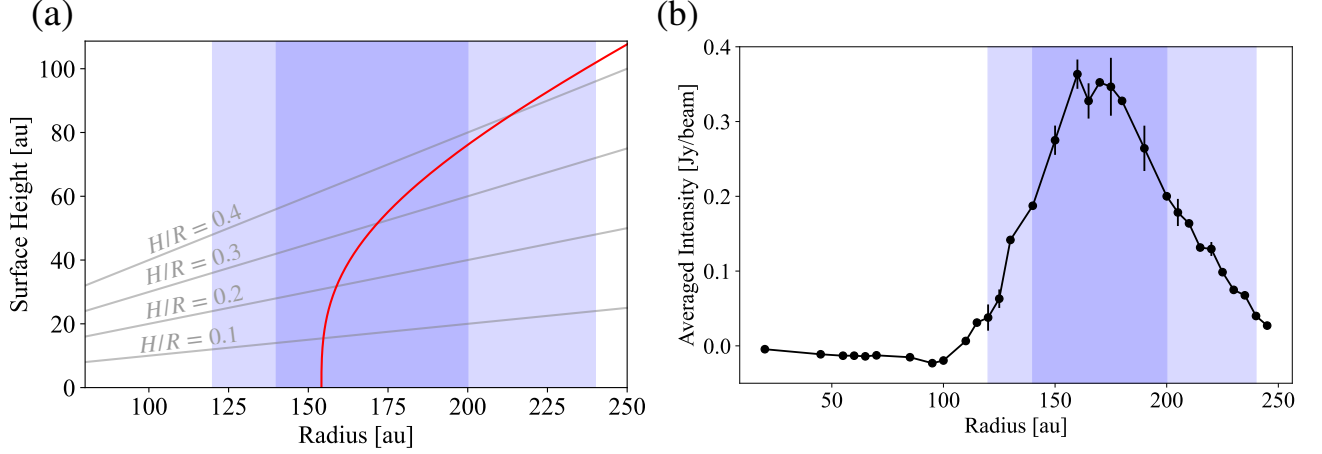


Figure 3.7: (a): Estimated disk surface height profile of HD142527 represented by Equation (3.6) (red line). (b): The azimuthally averaged intensity profile of the dust continuum emission observed with ALMA Band 9 (Casassus et al. 2015). Due to the strong non-axisymmetric features of this object, averaging was limited to a range of $\pm 30^\circ$ around the northern direction. The error bars represent the standard deviation of the intensity at each radius. The dark blue shaded region indicates areas with an intensity greater than half of the peak intensity, while the light blue shaded region corresponds to intensities greater than one-tenth of the peak intensity. The shaded regions in panel (a) follow the same definitions.

radius. This suggests that the effect of pressure bumps may be weak, or that other factors may be causing radial diffusion.

3.5 Discussion

3.5.1 Strengths of my Model

My new approach successfully constrains the geometric structure of both the outer and inner disks in more detail through the modeling of horizon and shadow boundaries.

Determination of the horizon is particularly important for constraining the central position (c_{ox}, c_{oy}) and the inner edge radius R_e of the outer disk. The horizon exhibits an elliptical shape, and the orientation and length of its major axis are dependent on the central position but not strongly correlated with the surface height of the outer disk. On the other hand, the vertical location of the scattering surface of the outer disk (H_c, α) influences the length of the minor axis of the horizon. However, this dependency is very small, making it difficult to precisely determine these parameters. Fortunately, these parameters have a significant impact on the shape of the shadow boundaries, enabling better constraints through the modeling of shadow boundaries.

Earlier studies, especially the shadow analysis by Min et al. (2017), used a shadow model that did not consider the thickness of the inner disk. The model calculated the width of the shadow using the PA and i values of the inner disk and the fixed aspect ratio of the outer disk. However, this led to problems with degeneracy in the combination of PA and i values. To improve upon this, I incorporated the aspect ratio of the inner disk into the model and also

integrated the information of shadow width. This improvement has resolved the degeneracy in PA and i of the inner disk, enabling a more accurate estimation of the disk structure.

3.5.2 Limitations of my Model and its Improvement Strategies

In my present model, the disk height converges to $H \propto R$ at large radii. However, some disks have a flared shape. This means that the model could be improved so that it can be used in these situations. The following formula for the location of the scattering surface is such an example of this:

$$R(H) = R_e \left(1 + \left(\frac{R_c^\beta |H|}{R_e^\beta H_c} \right)^{\frac{\alpha}{\beta}} \right)^{\frac{1}{\alpha}}, \quad (3.21)$$

$$\frac{dR}{dH} = \text{sign}(H) \frac{R_c^\alpha R_e^{1-\alpha}}{\beta H_c^{\frac{\alpha}{\beta}}} |H|^{\frac{\alpha}{\beta}-1} \left(1 + \left(\frac{R_c^\beta |H|}{R_e^\beta H_c} \right)^{\frac{\alpha}{\beta}} \right)^{\frac{1}{\alpha}-1}. \quad (3.22)$$

In these equations, the disk surface asymptotically approaches $H = H_c(R/R_c)^\beta$ in the outer regions. The parameter α adjusts the asymptotic rate in the radial direction, while β controls the flaring index.

Since the parameters α and β are correlated, it may be challenging to determine them accurately. To address this issue, incorporating a curve representing the outer edge of the outer disk into the model may improve the accuracy of parameter estimation. In such a strategy, the outer edge radius R_{out} is treated as a free parameter and is expressed by the following curve:

$$\begin{aligned} \mathbf{C}_{\text{out}} &= \mathbf{S}_o(\phi_o; R_{\text{out}}) \\ &= R_{\text{out}}(\cos(\phi_o)\hat{\mathbf{l}}_o + \sin(\phi_o)\hat{\mathbf{m}}_o) + H_{\text{out}}\hat{\mathbf{n}}_o + \mathbf{c}_o, \end{aligned} \quad (3.23)$$

where H_{out} is the surface height of the outer disk at radius R_{out} . However, careful consideration will be required when such a model with outer edge is applied to a disk manifesting significant non-axisymmetric structure.

My present model assumes that the vertical structure of the inner disk is symmetric with respect to the mid-plane. However, global magnetohydrodynamic (MHD) simulations by Bai (2017) showed that asymmetric disk winds could develop on both sides of the disk. Because of this, the vertical structures might be different between the upper and lower layers, and the inner disk may cast more complex shadows onto the outer disk. In light of this, the current model may not fully reproduce the shape and position of the shadows, implying the need for future refinements. To address such asymmetries, it may be necessary to use three-dimensional radiative transfer simulations, enabling comparisons with observations not only in terms of geometry but also in terms of intensity.

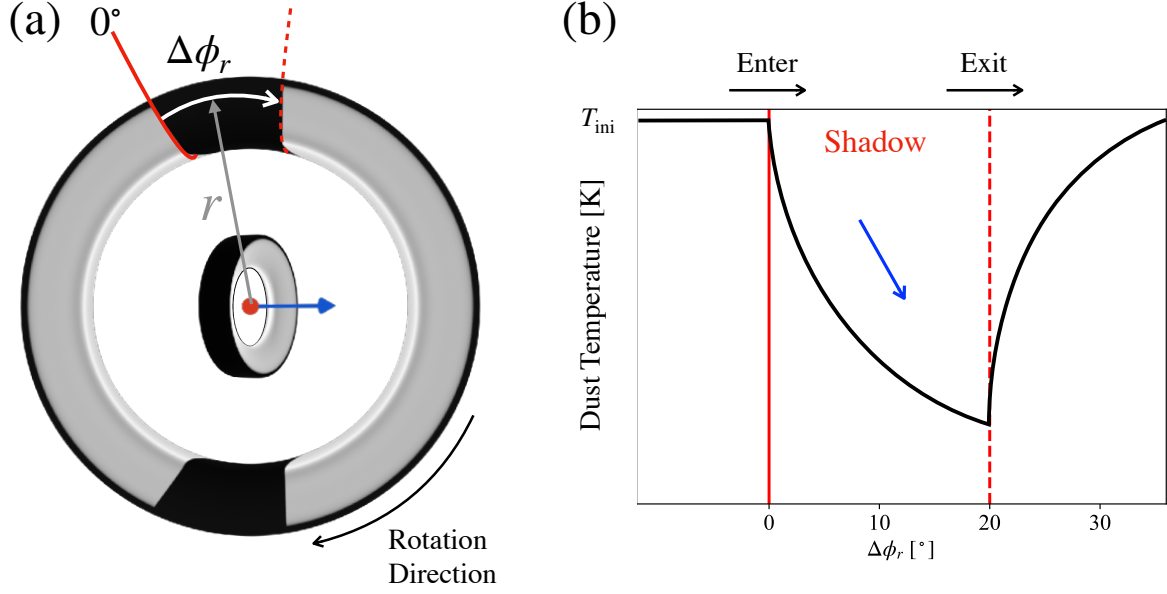


Figure 3.8: (a) Three-dimensional conceptual diagram of the dust scattering surface in the disk with the misaligned inner disk. The red and blue vectors represent the normal vector of the outer disk and the inner disk, respectively. This system is considered to have a clockwise rotation. The red curve represents the boundary between illuminated and shadowed areas. (b) Conceptual plot of the azimuthal temperature profile of the disk from the left model. The horizontal axis represents the position angle $\Delta\phi_r$ at a radius r , and is based on the shadow boundary where dust is entering.

3.5.3 Measuring the Cooling Time of Material in the Outer Disk

In the shadowed region, stellar heating is reduced, leading to a temperature drop due to the cooling of dust particles. By comparing the three-dimensional spatial distribution of shadows estimated from the present model with the temperature distribution of the disk derived from sub-millimeter ALMA observations, it becomes possible to estimate the cooling timescale of the dust. Notably, if the cooling time is significantly shorter than the Keplerian timescale, the gas ceases to behave adiabatically, and the stabilizing buoyancy force—normally acting to suppress vertical motions—becomes ineffective. This absence of stabilizing buoyancy is a key condition for the development of the Vertical Shear Instability (VSI) (Flock et al. 2020, Fukuhara et al. 2023). VSI is considered one of the mechanisms that drive turbulence, which in turn plays a crucial role in planetesimal formation by enhancing dust growth.

The primary objective of this section is to discuss the cooling timescale of dust as it enters shadowed regions within the disk. For this purpose, the temperature distribution was calculated based on the intensity distribution of dust continuum emission observed with ALMA, and the results were compared with the three-dimensional shadow region derived from my method. In this comparison, the spatial offset between the shadow region and the temperature drop detected by ALMA was analyzed in terms of azimuthal displacement (see Figure 3.8). By normalizing this displacement using the Keplerian angular velocity at the corresponding location, the cooling

process was modeled as the following temporal evolution of temperature:

$$T(t) = T_{\text{tar}} + (T_{\text{ini}} - T_{\text{tar}}) \exp\left(-\frac{t}{t_{\text{cool}}}\right), \quad (3.24)$$

where T_{ini} represents the initial temperature before the shadow passes, T_{tar} is the final equilibrium temperature caused by the shadow, and t_{cool} denotes the cooling timescale.

The dust cooling process in the disk of HD142527, with a stellar mass of $2.2 M_{\odot}$ (Verhoeff et al. 2011), has been analysed by comparing the observed dust temperature distribution with the shadow regions derived from the proposed model. Figure 3.9 shows the azimuth-radius distribution of the brightness temperature derived from the ALMA Band 9 dust continuum emission. The horizontal axis represents the azimuthal angle, the vertical axis the radius, and the color scale the brightness temperature. The shadowed regions identified from the OIR images are marked by black filled areas, while the dashed black line indicates the radius of 180 au where the cooling analysis was performed. To quantify the cooling process, the azimuthal profile of the dust temperature was extracted at a fixed radius of 180 au and converted from angular displacement to time units using the Keplerian angular velocity $\Omega_K = 1.2 \times 10^{-10} \text{ rad/s}$ (Figure 3.10). The red shaded region in Figure 3.10 represents the range of cooling models that adequately describe the data, corresponding to cooling timescales of 30 – 100 yr. The results revealed that the cooling time within the shadowed region is significantly shorter than the Keplerian orbital period of ~ 1500 yr, suggesting that the shadows generate local temperature gradients that may trigger the VSI.

For HD100453, no significant azimuthal intensity decrease was observed, so no analysis was performed. Currently, ALMA data are only available up to band 7, but observations at

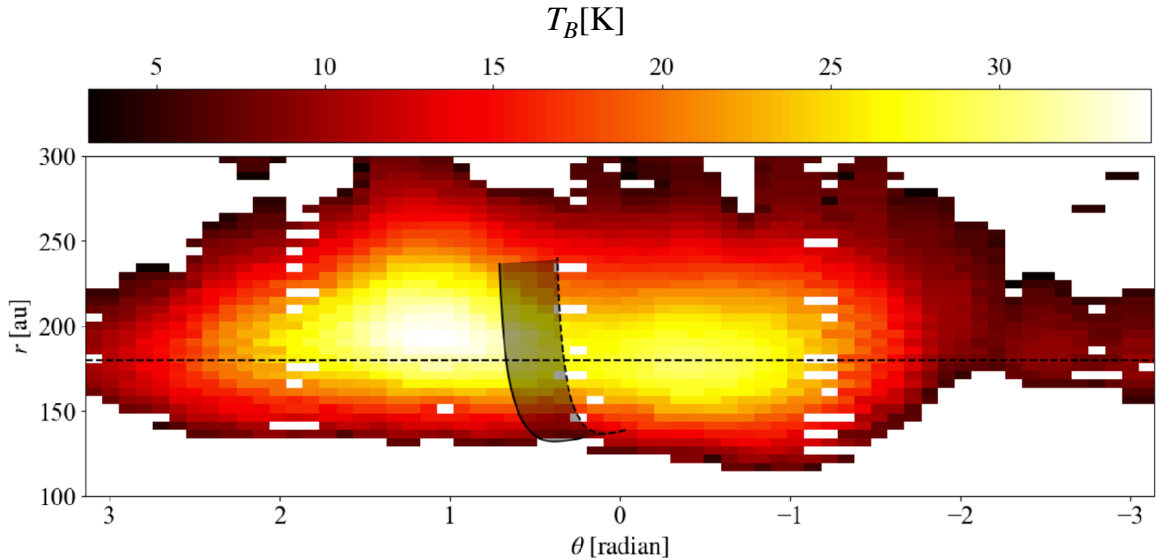


Figure 3.9: Temperature distribution of the disk around HD142527. The horizontal axis represents the azimuthal angle, while the vertical axis corresponds to the radial distance from the central star. The color scale indicates the brightness temperature derived from the dust continuum emission. The black filled regions denote shadowed areas inferred from the three-dimensional shadow model, and the dashed black line marks the radius of 180 au.

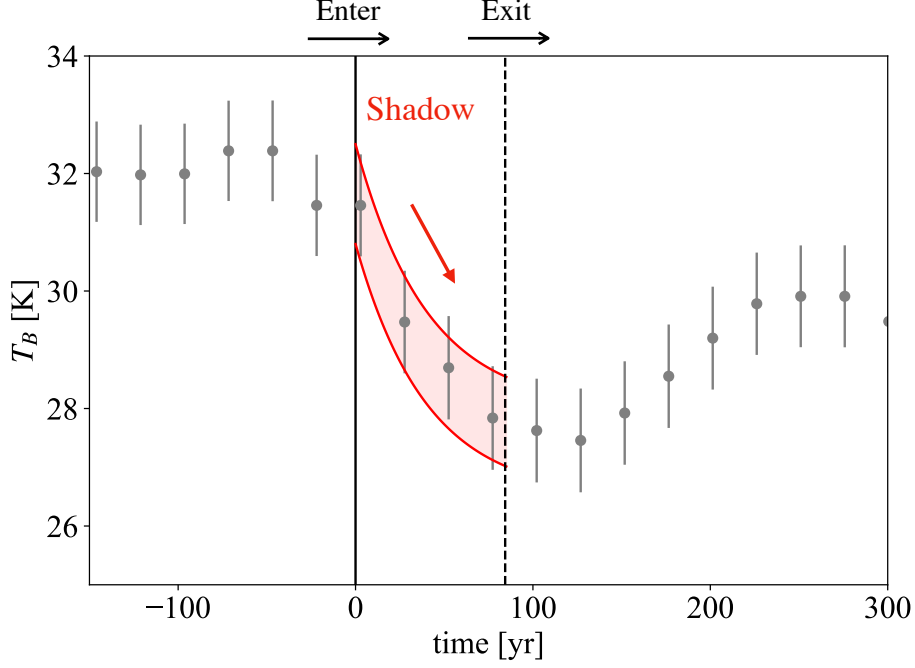


Figure 3.10: Time-temperature profile of dust cooling at a radius of 180 au. The vertical axis represents the brightness temperature of the dust continuum emission, while the horizontal axis corresponds to the azimuthal angle. Error bars are calculated through error propagation from the rms of the observational image. The red shaded region indicates the range of cooling models that fit the data, with cooling timescales ranging from 30 to 100 years.

higher frequencies may reveal such dips in the future. At higher frequencies, observations are advantageous for detecting temperature distributions not only because they offer higher spatial resolution to capture intensity variations caused by temperature drops, but also because thermal emission from optically thick dust becomes more prominent.

Multi-epoch observations by [Pinilla et al. \(2018\)](#) show that the shadows within the disk associated with J1604 exhibit variations on short timescales, on the order of a week. A comparison of OIR images with sub-millimeter images in different epochs carries a risk of misaligning the actual spatial relationship between the shadows and the temperature distribution. Such misalignment can result in incorrect estimations of cooling times. In order to address this issue in the context of cooling time measurements, it is necessary to either select objects that exhibit sufficiently long shadow variability timescales or to conduct simultaneous observations at OIR and sub-millimeter wavelengths.

3.5.4 Outlook Based on the New Information Provided by my Method

The modeling approach developed in this study, along with the obtained results, provides a foundation for further analyses and discussions. This section discusses possible future research directions and the physical mechanisms that may be explored.

Further Constraints on Inner Disk Geometry

The information estimated from the present model will facilitate more precise characterization of dust properties through SED analysis. By employing the orientation (i_i, PA_i) and aspect ratio h_r of the inner disk as fixed parameters in radiative transfer calculation, comparisons of observed SED enable detailed analysis of the vertical structure of dust density, size, and temperature within the inner disk.

According to Ueda et al. (2019), the accumulation of dust at the dead-zone inner edge can significantly alter the disk's vertical structure, forming a tall, optically thick dust wall. This structure casts an extended shadow on the outer disk, spanning several astronomical units beyond the edge, as shown in their radiative transfer simulations (e.g., their Figure 13). The maximum aspect ratio h/r derived from my modeling in this study may reflect the height of such a dust wall at the dead-zone inner edge. By utilizing this inferred vertical structure, it becomes possible to quantitatively constrain the location and geometric properties of the dead-zone inner edge. This, in turn, provides new insights into the thermal and dynamical processes that govern planetesimal formation and evolution in the innermost regions of protoplanetary disks.

Furthermore, if dust coupled with gas is blown away by the MHD wind and becomes optically thick (Miyake et al. 2016), it may affect the width of the shadow. Distinguishing between the actual width of the inner disk and the influence of the wind is challenging, requiring multi-epoch observations, such as simultaneous measurements of the velocity field of the inner disk and variations in the shadow shape.

Scattering Angles and Phase Function on the Outer Disk Surface

The model's capacity to precisely estimate the position of the stellar position and the surface height distribution of the outer disk facilitates the determination of the scattering angle at each position, as illustrated in Figure 3.11. The following equation calculates the scattering angle:

$$\theta_{\text{scat}} = \arccos \left(\frac{\mathbf{S}_o - \mathbf{c}_i}{|\mathbf{S}_o - \mathbf{c}_i|} \cdot \hat{\mathbf{e}}_z \right). \quad (3.25)$$

By comparing this scattering angle with the total intensity, I can derive the dust phase function (Ginski et al. 2023). The phase function represents the scattering characteristics of light by dust particles, and it allows us to estimate the size, shape, and composition of the dust from the observed scattered light.

In sub-sections 3.3 and 3.4, I use polarized intensity for the analysis. Further observational studies are necessary to advance this direction, in particular the imaging of total intensity, though this is technically demanding.

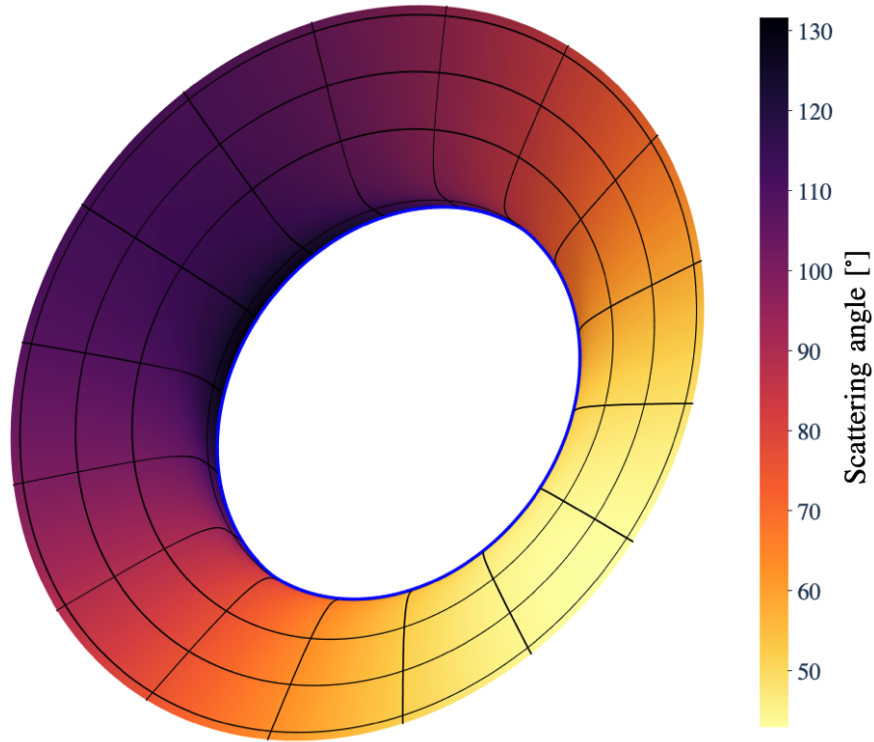


Figure 3.11: Scattering angle of the dust on the outer disk surface for the case of HD100453 whose disk parameters are derived from the modeling in section 3.3.

3.6 Conclusions

This study proposes a novel method for inferring the three-dimensional geometries of a transition disk system that consists of inner and outer disk. The method analyses the shadows cast by misaligned inner disks onto outer disks and the horizon on the outer disk’s surface in OIR scattered light images.

The proposed method offers several improvements over previous approaches, enabling the extraction of critical information about disk systems. It allows for the determination of the orientation (i_i , PA_i) and maximum aspect ratio h_r of inner disks, as well as the surface height profile of the outer disk (R_e , H_e , α). Additionally, the method resolves parameter degeneracies that were challenging in earlier methods, particularly the interdependence between the orientation of the inner disk and the thickness of the outer disk.

As a proof of concept, this method was applied to the HD100453 and HD142527 systems, providing significant insights into the geometrical and physical characteristics of their inner and outer disks. Both inner disks were found to have a misalignment angle of $> 50^\circ$ relative to the outer disks. This substantial misalignment highlights the potential influence of an additional companion in the disk system, as suggested by prior studies. The maximum aspect ratio of the inner disk (> 0.15) suggests the presence of an optically thick structure at the inner edge. This structure may be due to a “dust wall” formed by dust accumulation at the inner edge of the dead zone, or it could be influenced by dust lifted by a disk wind. The height profile of

the outer disk, on the other hand, was characterized by rapid convergence, indicative of dust accumulation influenced by gas pressure maxima.

This chapter establishes the methodological foundation for indirectly detecting and quantifying the influence of a misaligned inner disk on the outer disk. As demonstrated in Chapter 2, the presence of an inner disk can be inferred from asymmetries in the velocity field and temperature distribution of the outer disk, but these signatures alone are insufficient to directly constrain the inner disk’s geometry. In contrast, this chapter focuses on near-infrared scattered light images, analyzing shadows and the surface structure of the outer disk to reconstruct the relative spatial configuration between the inner and outer disks.

By treating the problem as an inverse one—inferring the geometry of the inner disk based on observational features of the outer disk—this approach enables quantitative assessment of the dynamical and thermal coupling between the two components, even when the inner disk itself cannot be spatially resolved. The framework developed here provides a crucial step toward the integrative diagnostic discussions in the following chapter and offers a basis for designing future observational strategies to identify and characterize misaligned disk systems.

Software: *astropy*, *Numpy*, *Matplotlib*, *emcee*

3.7 Acknowledgment

This work used the SPHERE Data Centre, jointly operated by OSUG/IPAG (Grenoble), PYTHEAS/LAM/CESAM (Marseille), OCA/Lagrange (Nice), Observatoire de Paris/LESIA (Paris), and Observatoire de Lyon. We thank Myriam Benisty for providing us with access to the J-band data from [Benisty et al. \(2017\)](#).

Appendix 3.1: Surface Height Profile of The Outer Disk

In this study, the surface height profile of the outer disk is not derived from first principles or physical models. Instead, it is represented by an empirical function adopted for the purpose of this analysis, as given by the following equation:

$$H(R) = H_c \left(\frac{R^\alpha - R_e^\alpha}{R_c^\alpha - R_e^\alpha} \right)^{\frac{1}{\alpha}}. \quad (3.26)$$

This function is chosen so that it asymptotically approaches $H \propto R$ at sufficiently large distances from the disk center ($R \gg R_e$). The parameter α plays a crucial role in determining the rate at which the function approaches its asymptotic behavior. When $\alpha = 2$, the function takes the form of a hyperbolic curve. Specifically, larger values of α cause the function to converge to its asymptotic form more rapidly, whereas smaller values result in a longer scale for the function to reach the asymptotic form. For the calculation of the horizon, Equation (3.26) was rearranged into the inverse function of H , $R(H)$, as shown in Equation (3.6).

This definition is also applied in the shadow boundary model described in Section 3.2.2, and is considered a sufficient approximation in the relatively inner regions where the shadow is visible. For example, if the aspect ratio of the outer disk surface has a convex maximum at a certain radius, regions beyond that point would not receive light, and consequently, no scattered light would be observable.

Appendix 3.2: Solutions of the Shadow Boundary Curves

The shadow boundary curves are obtained by solving the following equation with respect to ϕ_o , ϕ_i , r :

$$\mathbf{S}_o - \mathbf{S}_i = R(H)(\cos(\phi_o)\hat{\mathbf{l}}_o + \sin(\phi_o)\hat{\mathbf{m}}_o) + H\hat{\mathbf{n}}_o - r(\cos(\phi_i)\hat{\mathbf{l}}_i + \sin(\phi_i)\hat{\mathbf{m}}_i) \mp h_r r \hat{\mathbf{n}}_i + \delta \mathbf{c} = 0. \quad (3.27)$$

Expressing this equation using the orthonormal basis $\hat{\mathbf{l}}_i$, $\hat{\mathbf{m}}_i$, and $\hat{\mathbf{n}}_i$, which represents the inner disk, it can be written as follows:

$$\begin{aligned} \mathbf{S}_o - \mathbf{S}_i &= (R(H) \cos(\phi_o) - r \cos(\phi_i) + \delta \mathbf{c} \cdot \hat{\mathbf{l}}_i) \hat{\mathbf{l}}_i \\ &\quad + (R(H) \sin(\phi_o) \hat{\mathbf{m}}_o \cdot \hat{\mathbf{m}}_i + H \hat{\mathbf{n}}_o \cdot \hat{\mathbf{m}}_i - r \sin(\phi_i) + \delta \mathbf{c} \cdot \hat{\mathbf{m}}_i) \hat{\mathbf{m}}_i \\ &\quad + (R(H) \sin(\phi_o) \hat{\mathbf{n}}_o \cdot \hat{\mathbf{m}}_i + H \hat{\mathbf{n}}_o \cdot \hat{\mathbf{n}}_i \mp h_r r + \delta \mathbf{c} \cdot \hat{\mathbf{n}}_i) \hat{\mathbf{n}}_i \\ &= 0. \end{aligned} \quad (3.28)$$

By combining the three equations through inner product calculations, we can eliminate the variables r and ϕ_i , ultimately deriving a quartic equation that depends solely on $x = \sin(\phi_o)$. This equation takes the general form:

$$a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0, \quad (3.29)$$

where a_4, a_3, a_2, a_1 , and a_0 are coefficients determined by the parameters present in the original equations. These coefficients encapsulate the relationships between the geometric and physical parameters of the system, and their specific forms are given by:

$$a_4 = b_2^2, \quad (3.30)$$

$$a_3 = 2b_2b_1, \quad (3.31)$$

$$a_2 = b_1^2 + 2b_2b_0 + 4R^2(\delta\mathbf{c} \cdot \hat{\mathbf{l}}_i)^2, \quad (3.32)$$

$$a_1 = 2b_1b_0, \quad (3.33)$$

$$a_0 = b_0^2 - 4R^2(\delta\mathbf{c} \cdot \hat{\mathbf{l}}_i)^2. \quad (3.34)$$

Here, the intermediate variables b_2 , b_1 and b_0 are expressed in terms of the original parameters as follows:

$$b_2 = R^2 \left((\hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}_o)^2 - \left(\frac{\hat{\mathbf{m}}_i \cdot \hat{\mathbf{n}}_o}{h_r} \right)^2 - 1 \right), \quad (3.35)$$

$$b_1 = 2R\mathbf{H}_\delta \cdot \left((\hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}_o) \hat{\mathbf{m}}_i + \frac{\hat{\mathbf{m}}_i \cdot \hat{\mathbf{n}}_o}{h_r^2} \hat{\mathbf{n}}_i \right), \quad (3.36)$$

$$b_0 = R^2 + (\delta\mathbf{c} \cdot \hat{\mathbf{l}}_i)^2 + (\mathbf{H}_\delta \cdot \hat{\mathbf{m}}_i)^2 - \left(\frac{\mathbf{H}_\delta \cdot \hat{\mathbf{m}}_i}{h_r} \right)^2, \quad (3.37)$$

with $\mathbf{H}_\delta = H\hat{\mathbf{n}}_o + \delta\mathbf{c}$.

The quartic equation yields four solutions $\mathbf{C}_{\text{Sha}} = \mathbf{S}_o(H; \phi_{o,\text{Sha}}(H))$, which can be classified using Equation (3.28). Specifically, the sign of $\cos(\phi_o)$ distinguishes the two shadows, while the sign preceding h_r determines whether the boundary is made by the front side surface or the back side surface of the inner disk (see Figure 3.1).

Appendix 3.3: Corner Plot of the MCMC Results

The corner plots of the resulting marginal posterior distributions are shown in Figures 3.12 and 3.13.

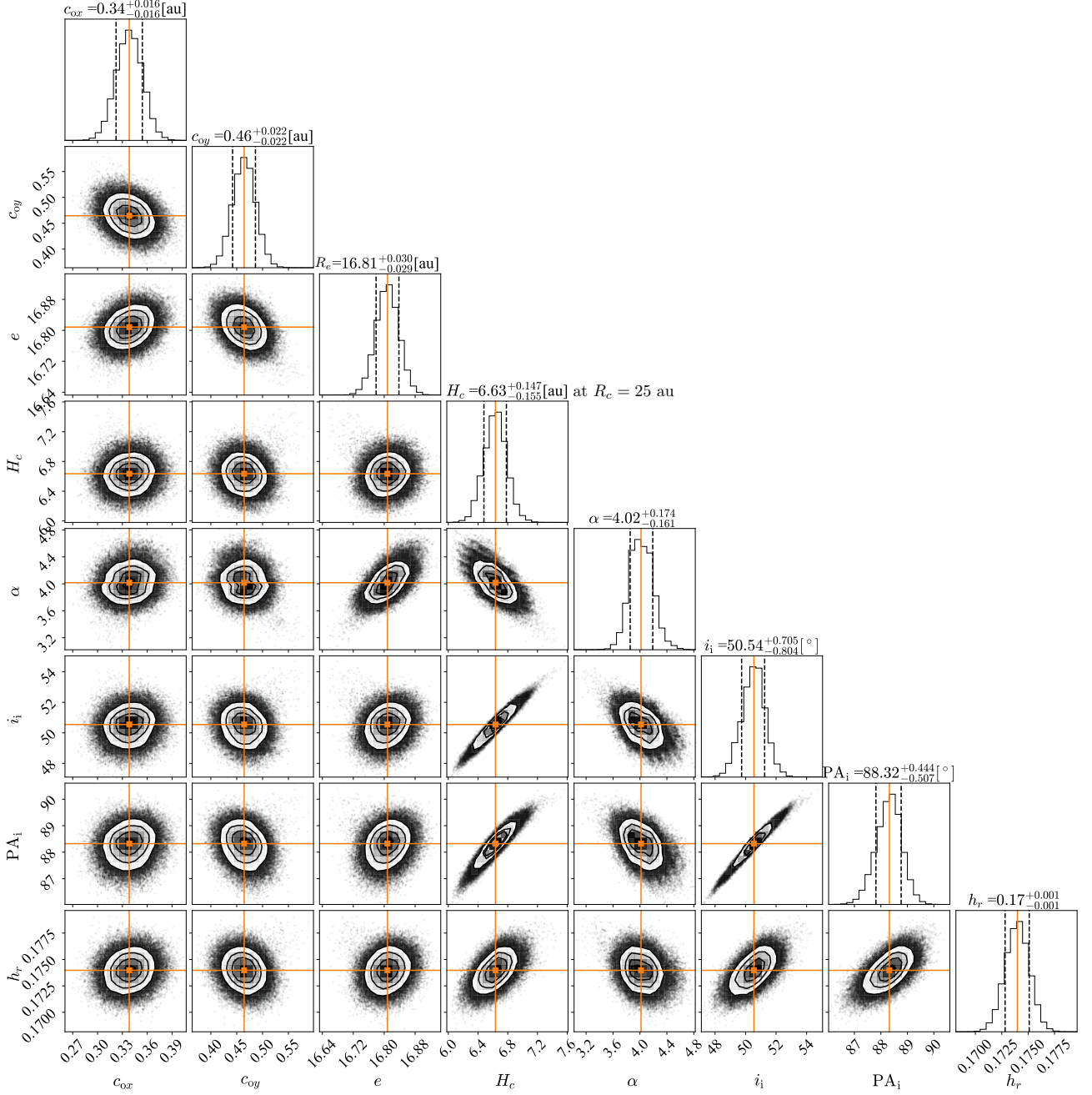


Figure 3.12: MCMC corner plot represents the results of my model fitting for HD100453. The values and uncertainties shown in the panel titles correspond to the median and the 16th and 84th percentiles, respectively.

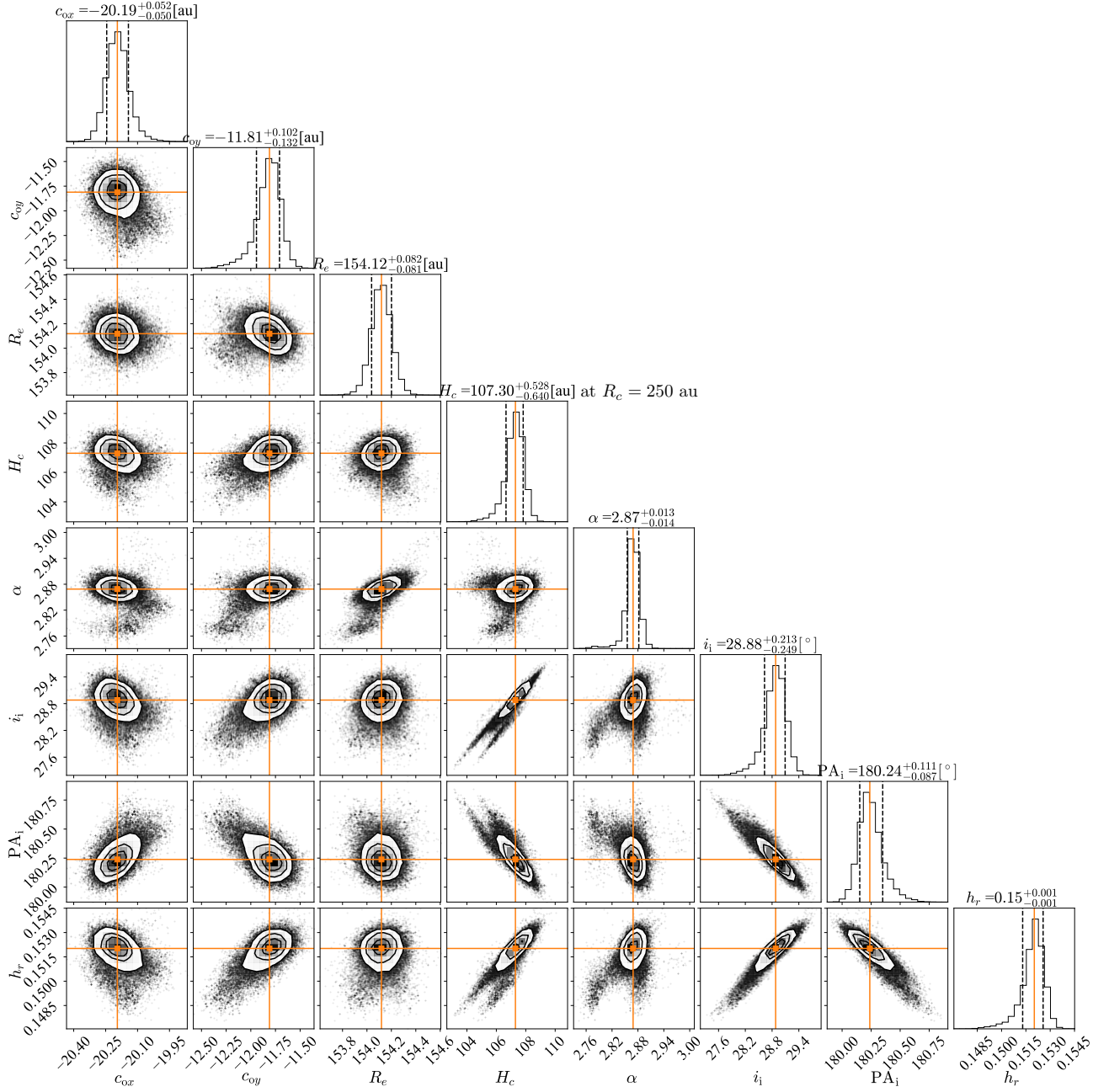


Figure 3.13: MCMC corner plot represents the results of my model fitting for HD142527. The values and uncertainties shown in the panel titles correspond to the median and the 16th and 84th percentiles, respectively.

Chapter 4

Summary

4.1 Summary of this thesis

This study investigates transition disks with inner disks in their innermost regions, focusing on features resulting from inner disk misalignment through the analysis of optical, infrared, and millimeter-wave data. Protoplanetary disks, composed of gas and dust surrounding young stars, are believed to evolve through the coagulation of dust particles into kilometer-sized planetesimals, which subsequently form planets. Understanding the structure and evolution of these disks is essential for elucidating the mechanisms of planet formation. Transition disks, characterized by a lack of significant near-infrared excess in their spectral energy distribution (SED), were traditionally interpreted as having depleted dust in their inner regions. However, recent imaging observations have revealed the presence of inner disks within these gaps, prompting a reevaluation of this interpretation.

This dissertation investigates the structure and kinematics of transition disks, with a particular focus on their inner and outer disk components. By utilizing high-resolution observational data and developing novel analytical methods, this work provides new insights into the physical and geometric properties of disk systems, especially those with inner disk misalignments. The key findings of this study are summarized below.

Chapter 2: ALMA Band 6 high-resolution observations of the transition disk around SY Cha

Using ALMA Band 6 observations, I analyzed the transition disk surrounding SY Cha. The analysis of both the dust continuum emission and the $J = 2-1$ line emission of CO isotopologues

revealed the following:

- The 225 GHz dust continuum emission exhibits a ring structure at a radius of ~ 100 au, accompanied by a central point source. Visibility-domain analysis indicated faint extended emission beyond the outer ring and a weak north-east brightness asymmetry. The total dust mass was estimated to be $1.0 \times 10^{-4} M_{\odot}$.
- Emissions of ^{12}CO , ^{13}CO , and C^{18}O were detected. While ^{13}CO emission is concentrated near the dust ring, ^{12}CO emission was detected inside the ring, suggesting the presence of residual gas within the dust gap. The total gas mass was estimated to be $2.2 \times 10^{-4} M_{\odot}$, with potential increases by factors of 10–100 if CO depletion is taken into account.
- The moment 1 map of CO emission revealed Keplerian rotation around a $0.78 M_{\odot}$ star, with velocity distortions near the star. These distortions could result from radial inflow or a warped inner disk. In the latter case, the inner disk may be inclined by $70\text{--}90^{\circ}$, potentially causing asymmetric illumination and weak asymmetry in the outer disk.

Chapter 3: Shadow-Based Framework for Estimating Transition Disk Geometries

Direct observations of inner disks remain challenging with current observational facilities due to their small angular sizes and the intense brightness of central stars. These limitations hinder the ability to resolve the physical and geometric structures of inner disks in detail, particularly in systems with highly misaligned inner disks. To overcome these challenges, this chapter introduces a new methodology to infer the three-dimensional geometries of transition disks. The method analyzes shadows cast by misaligned inner disks onto outer disks and the horizon on the outer disk surface in optical and infrared scattered light images. Key advancements of this method include:

- Determination of the inner disk orientation (i_i , PA_i), maximum aspect ratio (h_r), and the outer disk height profile (R_e , H_c , α).
- Resolution of parameter degeneracies, particularly between the inner disk orientation and outer disk thickness, which were challenging in previous methods.
- Application of the method to HD100453 and HD142527 revealed inner disks misaligned by $> 50^{\circ}$ relative to the outer disks, likely caused by gravitational interactions with companions. The inner disks' aspect ratio (> 0.15) suggests an optically thick dust layer at the dead zone inner edge, while the outer disks show rapid height convergence due to dust accumulation at gas pressure maxima.

Inverse Diagnostics of Inner Disk Geometry Based on Observational Signatures

To investigate how misaligned inner disks influence their surrounding structures, we compiled observational diagnostics across multiple wavelengths, as illustrated in Figure 4.1. Each panel corresponds to a different observational approach, all of which highlight indirect yet distinctive signatures caused by the inclination of the unresolved inner disk. These signatures, when interpreted in a unified framework, provide powerful constraints on the geometric and thermal structure of both inner and outer disks.

First, the top-left panel presents a schematic representation of a disk system with a misaligned inner disk. The tilt of the inner disk relative to the outer disk’s midplane causes differential illumination across the outer disk surface. This geometric configuration is the origin of various asymmetries seen in observational data.

The top-right panel shows an optical/infrared (OIR) scattered light image, where shadows cast by the misaligned inner disk are observed. These shadows manifest as localized dips in brightness and are aligned with the orientation of the inner disk. Since the shadow geometry depends on the inclination, position angle, and thickness of the inner disk—as well as the flaring of the outer disk—scattered light imaging provides critical constraints on the three-dimensional structure of the system.

The bottom-left panel displays the velocity deviation map derived from molecular line observations (e.g., CO isotopologues). While the outer disk largely follows Keplerian rotation, the inner region shows a significant departure from the expected velocity field. Such deviations are interpreted as evidence of radial inflow or disk warping, both of which can be induced by a misaligned inner component. This makes molecular line kinematics a valuable tool for probing the dynamical influence of inner disks.

The bottom-right panel shows a millimeter continuum image highlighting azimuthal asymmetries in dust temperature. These asymmetries, such as dips or brightness enhancements, are likely the result of uneven stellar irradiation due to the shadowing effect of the inner disk. Since the dust temperature influences the observed brightness, spatial variations in intensity can be interpreted as thermal footprints of inner disk misalignment.

Together, these observational approaches—scattered light shadows, kinematic warps, and thermal asymmetries—form a complementary set of diagnostics. Even when the inner disk itself is unresolved, its geometric and physical properties can be inferred from its influence on the outer disk. Thus, multi-wavelength imaging provides a means of solving an inverse problem: constraining small-scale disk structures through large-scale observational signatures.

Importantly, the diagnostic features presented here—shadows in scattered light, kinematic warps in velocity maps, and azimuthal variations in millimeter continuum brightness—can be analyzed without resorting to full radiative transfer modeling with tools such as RADMC-3D (Dullemond et al. 2012). This allows for efficient extraction of key physical quantities, such as the relative inclination and position angle of the inner disk, its scale height, and the flaring

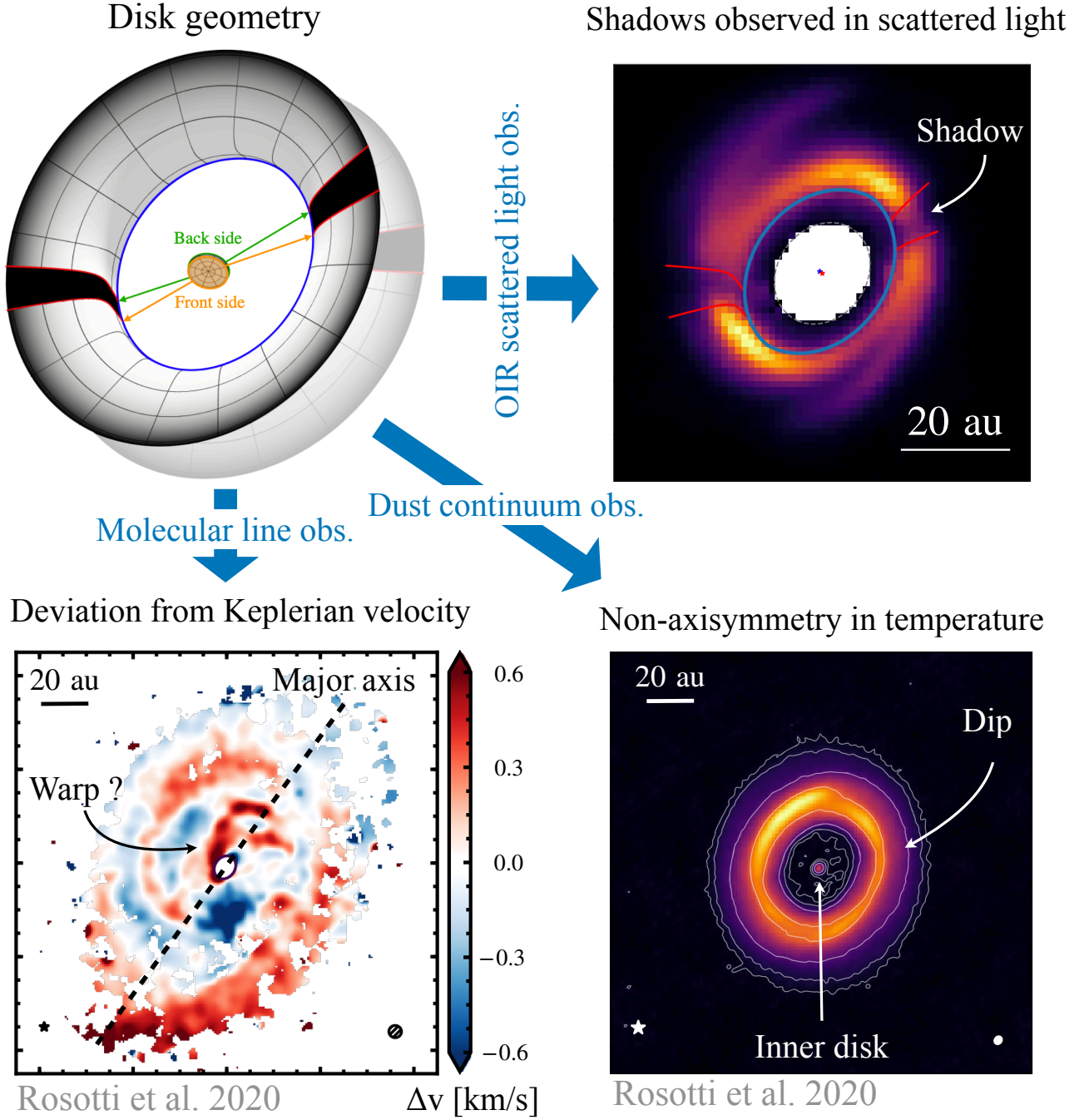


Figure 4.1: Schematic overview of the observational signatures produced by a misaligned inner disk, illustrated using the example of HD 100453. The misalignment leads to distinct features across multiple wavelength regimes: (Top left) A schematic diagram of a disk system with a misaligned inner component. (Top right) In optical and infrared (OIR) scattered light images, shadows are cast onto the outer disk, revealing the tilt and orientation of the inner disk. (Bottom left) Molecular line observations (adapted from Rosotti et al. 2020) show deviations from Keplerian rotation near the inner disk, which may indicate disk warping. (Bottom right) Dust continuum observations at millimeter wavelengths (adapted from Rosotti et al. 2020) display azimuthal temperature asymmetries in the outer disk, often associated with shadowed regions (“dips”) caused by the misaligned inner disk. Together, these multi-wavelength features offer a powerful pathway to constrain the geometry of the unresolved inner disk through indirect observations.

geometry of the outer disk, directly from observed images.

These quantities, once constrained, can then serve as fixed parameters or strong priors for subsequent radiative transfer calculations. Incorporating them into forward modeling enables more realistic simulations of the thermal and scattering behavior of disk systems. In this way, the methodology developed in this study lays the groundwork for a hybrid approach that combines observational diagnostics with physical modeling, offering a powerful path toward a more complete understanding of disk structure and its evolutionary implications.

A major advantage of this approach is that it significantly reduces the computational cost and complexity typically required for parameter exploration using radiative transfer codes. Without the need for time-consuming forward modeling at every step, it becomes possible to rapidly evaluate multiple systems or survey larger samples. This efficiency also facilitates quicker iteration between observation and interpretation, allowing for more responsive hypothesis testing and model refinement.

Finally, The observational signatures caused by a misaligned inner disk—such as shadows in scattered light, kinematic warps in velocity fields, and azimuthal brightness asymmetries in thermal emission—are not merely localized features, but rather direct manifestations of the dynamic and thermal interplay between the inner and outer disks. These features indicate that the inner disk exerts a significant influence on the outer disk’s temperature distribution, gas dynamics, and even potential sites of planet formation. Conversely, the structure and evolution of the outer disk, such as variations in mass accretion, can also shape the boundary conditions experienced by the inner disk. Therefore, rather than analyzing the inner and outer disks in isolation, a co-evolutionary framework is essential for capturing the mutual interactions and feedback processes that govern the overall evolution of the system.

4.2 Future works

Building upon the findings of this dissertation, several directions for future work are proposed to further investigate the physical and geometric properties of transition disks and their components:

Shadow Analysis of the Transition Disk around SY Cha

One promising direction for future research is the application of shadow analysis techniques to the transition disk around SY Cha. Currently, only low signal-to-noise ratio (SN) scattered light images of SY Cha have been reported, which are insufficient for detailed shadow analysis (Ginski et al. 2024). Therefore, follow-up observations to obtain high-SN scattered light images are essential. If shadows cast by the inner disk onto the outer disk are identified, the non-axisymmetric features observed in the outer disk could likely be attributed to localized cooling-induced temperature drops. Additionally, estimating the cooling timescale of the outer disk is expected to provide valuable insights into the thermal processes governing disk dynamics and

the factors driving non-axisymmetric structures.

Combining Measured Cooling Time with Numerical Simulations

Measuring cooling timescales from observations and integrating these measurements with numerical simulations could significantly enhance our understanding of the vertical structure of transition disks, including key parameters such as dust grain size and turbulence strength (Fukuhara & Okuzumi 2024).

Phase Function Analysis for Dust Properties

By deriving the scattering phase function from the estimated surface of the outer disk, it may be possible to constrain the dust size distribution within the scattering surface of the outer disk (Ginski et al. 2023). Comparing these results with the vertical dust distribution inferred from cooling time analyses could offer a more comprehensive understanding of dust properties, particularly the differences between the mid-plane and the scattering surface.

Vertical Structure of Inner Disks

The aspect ratio of the inner disk, as constrained by the methodology proposed in this dissertation, could serve as a crucial input for radiative transfer calculations and SED modeling. These approaches could further refine our understanding of the vertical structure of the inner disk, including its dust composition and thermal properties.

Bibliography

- ALMA Science Operations: Cycle 6 Pipeline Reference Manual. 2018, <https://almascience.org/processing/documents-and-tools/alma-science-pipeline-reference-manual-5-4>
- ALMA Science Pipeline User's Guide for CASA 5.4. 2018, <https://almascience.org/processing/documents-and-tools/alma-science-pipeline-users-guide-casa-5-4>
- Andre, P., Ward-Thompson, D., & Barsony, M. 1993, ApJ, 406, 122, doi: [10.1086/172425](https://doi.org/10.1086/172425)
- Andrews, S. M., Wilner, D. J., Zhu, Z., et al. 2016, ApJ, 820, L40, doi: [10.3847/2041-8205/820/2/L40](https://doi.org/10.3847/2041-8205/820/2/L40)
- Armitage, P. J. 2015, arXiv e-prints, arXiv:1509.06382, doi: [10.48550/arXiv.1509.06382](https://doi.org/10.48550/arXiv.1509.06382)
- Avenhaus, H., Quanz, S. P., Schmid, H. M., et al. 2017, AJ, 154, 33, doi: [10.3847/1538-3881/aa7560](https://doi.org/10.3847/1538-3881/aa7560)
- Baddour, N. 2009, J. Opt. Soc. Am. A, 26, 1767
- Bai, X.-N. 2017, in Astrophysics and Space Science Library, Vol. 445, Formation, Evolution, and Dynamics of Young Solar Systems, ed. M. Pessah & O. Gressel, 63, doi: [10.1007/978-3-319-60609-5_3](https://doi.org/10.1007/978-3-319-60609-5_3)
- Bai, X.-N., & Stone, J. M. 2013, ApJ, 769, 76, doi: [10.1088/0004-637X/769/1/76](https://doi.org/10.1088/0004-637X/769/1/76)
- . 2017, ApJ, 836, 46, doi: [10.3847/1538-4357/836/1/46](https://doi.org/10.3847/1538-4357/836/1/46)
- Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214, doi: [10.1086/170270](https://doi.org/10.1086/170270)
- Bayo, A., Rodrigo, C., Barrado Y Navascués, D., et al. 2008, A&A, 492, 277, doi: [10.1051/0004-6361:200810395](https://doi.org/10.1051/0004-6361:200810395)
- Beckwith, S. V. W., Sargent, A. I., Chini, R. S., & Guesten, R. 1990, AJ, 99, 924, doi: [10.1086/115385](https://doi.org/10.1086/115385)
- Belloche, A., Schuller, F., Parise, B., et al. 2011, A&A, 527, A145, doi: [10.1051/0004-6361/201015733](https://doi.org/10.1051/0004-6361/201015733)

- Benisty, M., Stolker, T., Pohl, A., et al. 2017, *A&A*, 597, A42, doi: [10.1051/0004-6361/201629798](https://doi.org/10.1051/0004-6361/201629798)
- Benisty, M., Dominik, C., Follette, K., et al. 2023, in *Astronomical Society of the Pacific Conference Series*, Vol. 534, *Protostars and Planets VII*, ed. S. Inutsuka, Y. Aikawa, T. Muto, K. Tomida, & M. Tamura, 605, doi: [10.48550/arXiv.2203.09991](https://doi.org/10.48550/arXiv.2203.09991)
- Bohn, A. J., Benisty, M., Perraut, K., et al. 2022, *A&A*, 658, A183, doi: [10.1051/0004-6361/202142070](https://doi.org/10.1051/0004-6361/202142070)
- Brown, J. M., Pontoppidan, K. M., van Dishoeck, E. F., et al. 2013, *ApJ*, 770, 94, doi: [10.1088/0004-637X/770/2/94](https://doi.org/10.1088/0004-637X/770/2/94)
- Canny, J. 1986, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, PAMI-8, 679, doi: [10.1109/TPAMI.1986.4767851](https://doi.org/10.1109/TPAMI.1986.4767851)
- Casassus, S., Marino, S., Pérez, S., et al. 2015, *ApJ*, 811, 92, doi: [10.1088/0004-637X/811/2/92](https://doi.org/10.1088/0004-637X/811/2/92)
- Casassus, S., Avenhaus, H., Pérez, S., et al. 2018, *MNRAS*, 477, 5104, doi: [10.1093/mnras/sty894](https://doi.org/10.1093/mnras/sty894)
- casatasks. 2020, <https://casadocs.readthedocs.io/en/v6.2.0/api/casatasks.html>
- Chiang, E. I., & Goldreich, P. 1997, *ApJ*, 490, 368, doi: [10.1086/304869](https://doi.org/10.1086/304869)
- Cornwell, T. J. 2008, *IEEE Journal of Selected Topics in Signal Processing*, 2, 793, doi: [10.1109/JSTSP.2008.2006388](https://doi.org/10.1109/JSTSP.2008.2006388)
- Cutri, R. M., Skrutskie, M. F., van Dyk, S., et al. 2003, *VizieR Online Data Catalog*, II/246
- Dauphas, N., & Chaussidon, M. 2011, *Annual Review of Earth and Planetary Sciences*, 39, 351, doi: [10.1146/annurev-earth-040610-133428](https://doi.org/10.1146/annurev-earth-040610-133428)
- de Gregorio-Monsalvo, I., Ménard, F., Dent, W., et al. 2013, *A&A*, 557, A133, doi: [10.1051/0004-6361/201321603](https://doi.org/10.1051/0004-6361/201321603)
- Dong, R., van der Marel, N., Hashimoto, J., et al. 2017, *ApJ*, 836, 201, doi: [10.3847/1538-4357/aa5abf](https://doi.org/10.3847/1538-4357/aa5abf)
- Dullemond, C. P., Juhasz, A., Pohl, A., et al. 2012, *RADMC-3D: A multi-purpose radiative transfer tool*, *Astrophysics Source Code Library*, record ascl:1202.015
- Dullemond, C. P., Küffmeier, M., Goicovic, F., et al. 2019, *A&A*, 628, A20, doi: [10.1051/0004-6361/201832632](https://doi.org/10.1051/0004-6361/201832632)

- Espaillat, C., Calvet, N., D’Alessio, P., et al. 2007, *ApJ*, 670, L135, doi: [10.1086/524360](https://doi.org/10.1086/524360)
- Espaillat, C., Furlan, E., D’Alessio, P., et al. 2011, *ApJ*, 728, 49, doi: [10.1088/0004-637X/728/1/49](https://doi.org/10.1088/0004-637X/728/1/49)
- Espaillat, C., Muzerolle, J., Najita, J., et al. 2014, in *Protostars and Planets VI*, ed. H. Beuther, R. S. Klessen, C. P. Dullemond, & T. Henning, 497–520, doi: [10.2458/azu_uapress_9780816531240-ch022](https://doi.org/10.2458/azu_uapress_9780816531240-ch022)
- Flock, M., Fromang, S., Turner, N. J., & Benisty, M. 2016, *ApJ*, 827, 144, doi: [10.3847/0004-637X/827/2/144](https://doi.org/10.3847/0004-637X/827/2/144)
- Flock, M., Turner, N. J., Nelson, R. P., et al. 2020, *ApJ*, 897, 155, doi: [10.3847/1538-4357/ab9641](https://doi.org/10.3847/1538-4357/ab9641)
- Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, *PASP*, 125, 306, doi: [10.1086/670067](https://doi.org/10.1086/670067)
- Fouqué, P., Chevallier, L., Cohen, M., et al. 2000, *A&AS*, 141, 313, doi: [10.1051/aas:2000123](https://doi.org/10.1051/aas:2000123)
- Francis, L., & van der Marel, N. 2020, *ApJ*, 892, 111, doi: [10.3847/1538-4357/ab7b63](https://doi.org/10.3847/1538-4357/ab7b63)
- Frasca, A., Biazzo, K., Lanzafame, A. C., et al. 2015, *A&A*, 575, A4, doi: [10.1051/0004-6361/201424409](https://doi.org/10.1051/0004-6361/201424409)
- Fukagawa, M., Tamura, M., Itoh, Y., et al. 2006, *ApJ*, 636, L153, doi: [10.1086/500128](https://doi.org/10.1086/500128)
- Fukagawa, M., Tsukagoshi, T., Momose, M., et al. 2013, *PASJ*, 65, L14, doi: [10.1093/pasj/65.6.L14](https://doi.org/10.1093/pasj/65.6.L14)
- Fukuhara, Y., & Okuzumi, S. 2024, *PASJ*, 76, 708, doi: [10.1093/pasj/psae042](https://doi.org/10.1093/pasj/psae042)
- Fukuhara, Y., Okuzumi, S., & Ono, T. 2023, *PASJ*, 75, 233, doi: [10.1093/pasj/psac107](https://doi.org/10.1093/pasj/psac107)
- Gaia Collaboration, Brown, A. G. A., Vallenari, A., et al. 2018, *A&A*, 616, A1, doi: [10.1051/0004-6361/201833051](https://doi.org/10.1051/0004-6361/201833051)
- . 2021, *A&A*, 649, A1, doi: [10.1051/0004-6361/202039657](https://doi.org/10.1051/0004-6361/202039657)
- Geers, V. C., Davis, L., Hales, C. A., et al. 2019, in *Astronomical Society of the Pacific Conference Series*, Vol. 521, *Astronomical Data Analysis Software and Systems XXVI*, ed. M. Molinaro, K. Shortridge, & F. Pasian, 366
- Ginski, C., Tazaki, R., Dominik, C., & Stolker, T. 2023, *ApJ*, 953, 92, doi: [10.3847/1538-4357/acdc97](https://doi.org/10.3847/1538-4357/acdc97)

- Ginski, C., Garufi, A., Benisty, M., et al. 2024, A&A, 685, A52, doi: [10.1051/0004-6361/202244005](https://doi.org/10.1051/0004-6361/202244005)
- Goodman, J., & Weare, J. 2010, Communications in Applied Mathematics and Computational Science, 5, 65, doi: [10.2140/camcos.2010.5.65](https://doi.org/10.2140/camcos.2010.5.65)
- Grady, C. A., Woodgate, B., Bruhweiler, F. C., et al. 1999, ApJ, 523, L151, doi: [10.1086/312270](https://doi.org/10.1086/312270)
- GRAVITY Collaboration, Perraut, K., Labadie, L., et al. 2019, A&A, 632, A53, doi: [10.1051/0004-6361/201936403](https://doi.org/10.1051/0004-6361/201936403)
- Hasegawa, Y., Haworth, T. J., Hoadley, K., et al. 2021, arXiv e-prints, arXiv:2112.02831. <https://arxiv.org/abs/2112.02831>
- Hashimoto, J., Muto, T., Dong, R., et al. 2021, ApJ, 911, 5, doi: [10.3847/1538-4357/abe59f](https://doi.org/10.3847/1538-4357/abe59f)
- Hayashi, C. 1981, Progress of Theoretical Physics Supplement, 70, 35, doi: [10.1143/PTPS.70.35](https://doi.org/10.1143/PTPS.70.35)
- Högbom, J. A. 1974, A&AS, 15, 417
- Hogg, D. W., Bovy, J., & Lang, D. 2010, arXiv e-prints, arXiv:1008.4686. <https://arxiv.org/abs/1008.4686>
- Hunziker, S., Schmid, H. M., Ma, J., et al. 2021, A&A, 648, A110, doi: [10.1051/0004-6361/202040166](https://doi.org/10.1051/0004-6361/202040166)
- Ishihara, D., Onaka, T., Kataza, H., et al. 2010, A&A, 514, A1, doi: [10.1051/0004-6361/200913811](https://doi.org/10.1051/0004-6361/200913811)
- Kanagawa, K. D., Hashimoto, J., Muto, T., et al. 2021, ApJ, 909, 212, doi: [10.3847/1538-4357/abdfc5](https://doi.org/10.3847/1538-4357/abdfc5)
- Kozai, Y. 1962, AJ, 67, 591, doi: [10.1086/108790](https://doi.org/10.1086/108790)
- Kretke, K. A., & Lin, D. N. C. 2007, ApJ, 664, L55, doi: [10.1086/520718](https://doi.org/10.1086/520718)
- Kretke, K. A., Lin, D. N. C., Garaud, P., & Turner, N. J. 2009, ApJ, 690, 407, doi: [10.1088/0004-637X/690/1/407](https://doi.org/10.1088/0004-637X/690/1/407)
- Kudo, T., Hashimoto, J., Muto, T., et al. 2018, ApJ, 868, L5, doi: [10.3847/2041-8213/aaeb1c](https://doi.org/10.3847/2041-8213/aaeb1c)
- Kuffmeier, M., Dullemond, C. P., Reissl, S., & Goicovic, F. G. 2021, A&A, 656, A161, doi: [10.1051/0004-6361/202039614](https://doi.org/10.1051/0004-6361/202039614)
- Kuhn, J. R., Potter, D., & Parise, B. 2001, ApJ, 553, L189, doi: [10.1086/320686](https://doi.org/10.1086/320686)

- Lada, C. J., & Wilking, B. A. 1984, *ApJ*, 287, 610, doi: [10.1086/162719](https://doi.org/10.1086/162719)
- Larson, R. B. 1969, *MNRAS*, 145, 271, doi: [10.1093/mnras/145.3.271](https://doi.org/10.1093/mnras/145.3.271)
- Lazareff, B., Berger, J. P., Kluska, J., et al. 2017, *A&A*, 599, A85, doi: [10.1051/0004-6361/201629305](https://doi.org/10.1051/0004-6361/201629305)
- Li, F., Cornwell, T. J., & de Hoog, F. 2011, *A&A*, 528, A31, doi: [10.1051/0004-6361/201015045](https://doi.org/10.1051/0004-6361/201015045)
- Lidov, M. L. 1962, *Planet. Space Sci.*, 9, 719, doi: [10.1016/0032-0633\(62\)90129-0](https://doi.org/10.1016/0032-0633(62)90129-0)
- Liu, H. B. 2019, *ApJ*, 877, L22, doi: [10.3847/2041-8213/ab1f8e](https://doi.org/10.3847/2041-8213/ab1f8e)
- Lovelace, R. V. E., Li, H., Colgate, S. A., & Nelson, A. F. 1999, *ApJ*, 513, 805, doi: [10.1086/306900](https://doi.org/10.1086/306900)
- Lynden-Bell, D., & Pringle, J. E. 1974, *MNRAS*, 168, 603, doi: [10.1093/mnras/168.3.603](https://doi.org/10.1093/mnras/168.3.603)
- Manara, C. F., Testi, L., Herczeg, G. J., et al. 2017, *A&A*, 604, A127, doi: [10.1051/0004-6361/201630147](https://doi.org/10.1051/0004-6361/201630147)
- Mangum, J. G., & Shirley, Y. L. 2015, *PASP*, 127, 266, doi: [10.1086/680323](https://doi.org/10.1086/680323)
- Marino, S., Perez, S., & Casassus, S. 2015, *ApJ*, 798, L44, doi: [10.1088/2041-8205/798/2/L44](https://doi.org/10.1088/2041-8205/798/2/L44)
- Marois, C., Lafrenière, D., Doyon, R., Macintosh, B., & Nadeau, D. 2006, *ApJ*, 641, 556, doi: [10.1086/500401](https://doi.org/10.1086/500401)
- Marsh, K. A., & Richardson, J. M. 1987, *A&A*, 182, 174
- Martin, R. G., Lubow, S. H., Nixon, C., & Armitage, P. J. 2016, *MNRAS*, 458, 4345, doi: [10.1093/mnras/stw605](https://doi.org/10.1093/mnras/stw605)
- Masset, F. S., Morbidelli, A., Crida, A., & Ferreira, J. 2006, *ApJ*, 642, 478, doi: [10.1086/500967](https://doi.org/10.1086/500967)
- Mayama, S., Hashimoto, J., Muto, T., et al. 2012, *ApJ*, 760, L26, doi: [10.1088/2041-8205/760/2/L26](https://doi.org/10.1088/2041-8205/760/2/L26)
- Mayama, S., Akiyama, E., Panić, O., et al. 2018, *ApJ*, 868, L3, doi: [10.3847/2041-8213/aae88b](https://doi.org/10.3847/2041-8213/aae88b)
- McMullin, J. P., Waters, B., Schiebel, D., Young, W., & Golap, K. 2007, in *Astronomical Society of the Pacific Conference Series*, Vol. 376, *Astronomical Data Analysis Software and Systems XVI*, ed. R. A. Shaw, F. Hill, & D. J. Bell, 127
- Min, M., Stolker, T., Dominik, C., & Benisty, M. 2017, *A&A*, 604, L10, doi: [10.1051/0004-6361/201730949](https://doi.org/10.1051/0004-6361/201730949)

- Miotello, A., Kamp, I., Birnstiel, T., Cleeves, L. C., & Kataoka, A. 2023, in *Astronomical Society of the Pacific Conference Series*, Vol. 534, *Protostars and Planets VII*, ed. S. Inutsuka, Y. Aikawa, T. Muto, K. Tomida, & M. Tamura, 501, doi: [10.48550/arXiv.2203.09818](https://doi.org/10.48550/arXiv.2203.09818)
- Miyake, T., Suzuki, T. K., & Inutsuka, S.-i. 2016, *ApJ*, 821, 3, doi: [10.3847/0004-637X/821/1/3](https://doi.org/10.3847/0004-637X/821/1/3)
- Montesinos, M., & Cuello, N. 2018, *MNRAS*, 475, L35, doi: [10.1093/mnrasl/sly001](https://doi.org/10.1093/mnrasl/sly001)
- Montesinos, M., Perez, S., Casassus, S., et al. 2016, *ApJ*, 823, L8, doi: [10.3847/2041-8205/823/1/L8](https://doi.org/10.3847/2041-8205/823/1/L8)
- Muto, T. 2011, *ApJ*, 739, 10, doi: [10.1088/0004-637X/739/1/10](https://doi.org/10.1088/0004-637X/739/1/10)
- Nealon, R., Cuello, N., Gonzalez, J.-F., et al. 2020a, *MNRAS*, 499, 3857, doi: [10.1093/mnras/staa2721](https://doi.org/10.1093/mnras/staa2721)
- Nealon, R., Dipierro, G., Alexander, R., Martin, R. G., & Nixon, C. 2018, *MNRAS*, 481, 20, doi: [10.1093/mnras/sty2267](https://doi.org/10.1093/mnras/sty2267)
- Nealon, R., Price, D. J., & Pinte, C. 2020b, *MNRAS*, 493, L143, doi: [10.1093/mnrasl/slaa026](https://doi.org/10.1093/mnrasl/slaa026)
- Neugebauer, G., Habing, H. J., van Duinen, R., et al. 1984, *ApJ*, 278, L1, doi: [10.1086/184209](https://doi.org/10.1086/184209)
- Nixon, C., King, A., & Price, D. 2013, *MNRAS*, 434, 1946, doi: [10.1093/mnras/stt1136](https://doi.org/10.1093/mnras/stt1136)
- Orihara, R., & Momose, M. 2025, arXiv e-prints, arXiv:2505.06044, doi: [10.48550/arXiv.2505.06044](https://doi.org/10.48550/arXiv.2505.06044)
- Orihara, R., Momose, M., Muto, T., et al. 2023, *PASJ*, 75, 424, doi: [10.1093/pasj/psad009](https://doi.org/10.1093/pasj/psad009)
- Owen, J. E., & Lai, D. 2017, *MNRAS*, 469, 2834, doi: [10.1093/mnras/stx1033](https://doi.org/10.1093/mnras/stx1033)
- Pinilla, P., Birnstiel, T., Ricci, L., et al. 2012, *A&A*, 538, A114, doi: [10.1051/0004-6361/201118204](https://doi.org/10.1051/0004-6361/201118204)
- Pinilla, P., Klarmann, L., Birnstiel, T., et al. 2016, *A&A*, 585, A35, doi: [10.1051/0004-6361/201527131](https://doi.org/10.1051/0004-6361/201527131)
- Pinilla, P., Benisty, M., de Boer, J., et al. 2018, *ApJ*, 868, 85, doi: [10.3847/1538-4357/aae824](https://doi.org/10.3847/1538-4357/aae824)
- Pinte, C., Teague, R., Flaherty, K., et al. 2023, in *Astronomical Society of the Pacific Conference Series*, Vol. 534, *Protostars and Planets VII*, ed. S. Inutsuka, Y. Aikawa, T. Muto, K. Tomida, & M. Tamura, 645, doi: [10.48550/arXiv.2203.09528](https://doi.org/10.48550/arXiv.2203.09528)
- Pontoppidan, K. M., Blake, G. A., van Dishoeck, E. F., et al. 2008, *ApJ*, 684, 1323, doi: [10.1086/590400](https://doi.org/10.1086/590400)

- Qi, C., D’Alessio, P., Öberg, K. I., et al. 2011, ApJ, 740, 84, doi: [10.1088/0004-637X/740/2/84](https://doi.org/10.1088/0004-637X/740/2/84)
- Rosenfeld, K. A., Chiang, E., & Andrews, S. M. 2014, ApJ, 782, 62, doi: [10.1088/0004-637X/782/2/62](https://doi.org/10.1088/0004-637X/782/2/62)
- Rosotti, G. P., Benisty, M., Juhász, A., et al. 2020, MNRAS, 491, 1335, doi: [10.1093/mnras/stz3090](https://doi.org/10.1093/mnras/stz3090)
- Sasi, N. M., & Jayasree, V. K. 2013, Engineering, 5, 326, doi: [10.4236/eng.2013.510B066](https://doi.org/10.4236/eng.2013.510B066)
- Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
- Sheikhnezami, S., Fendt, C., Porth, O., Vaidya, B., & Ghanbari, J. 2012, ApJ, 757, 65, doi: [10.1088/0004-637X/757/1/65](https://doi.org/10.1088/0004-637X/757/1/65)
- Siess, L., Dufour, E., & Forestini, M. 2000, A&A, 358, 593. <https://arxiv.org/abs/astro-ph/0003477>
- Siess, L., Forestini, M., & Dougados, C. 1997, A&A, 324, 556
- Soon, K.-L., Momose, M., Muto, T., et al. 2019, PASJ, 71, 124, doi: [10.1093/pasj/psz112](https://doi.org/10.1093/pasj/psz112)
- Suzuki, T. K., Ogihara, M., Morbidelli, A., Crida, A., & Guillot, T. 2016, A&A, 596, A74, doi: [10.1051/0004-6361/201628955](https://doi.org/10.1051/0004-6361/201628955)
- Tazaki, R., Tanaka, H., Okuzumi, S., Kataoka, A., & Nomura, H. 2016, ApJ, 823, 70, doi: [10.3847/0004-637X/823/2/70](https://doi.org/10.3847/0004-637X/823/2/70)
- Teague, R. 2020, richteague/keplerian_mask: Initial Release, 1.0, Zenodo, doi: [10.5281/zenodo.4321137](https://doi.org/10.5281/zenodo.4321137)
- Teague, R., & Foreman-Mackey, D. 2018, Bettermoments: A Robust Method To Measure Line Centroids, v1.0, Zenodo, doi: [10.5281/zenodo.1419754](https://doi.org/10.5281/zenodo.1419754)
- Thompson, A. R., Moran, J. M., & Swenson, George W., J. 2017, Interferometry and Synthesis in Radio Astronomy, 3rd Edition (Springer, Cham), doi: [10.1007/978-3-319-44431-4](https://doi.org/10.1007/978-3-319-44431-4)
- Tomida, K., Machida, M. N., Saigo, K., Tomisaka, K., & Matsumoto, T. 2010, ApJ, 725, L239, doi: [10.1088/2041-8205/725/2/L239](https://doi.org/10.1088/2041-8205/725/2/L239)
- Toomre, A. 1964, ApJ, 139, 1217, doi: [10.1086/147861](https://doi.org/10.1086/147861)
- Tsukagoshi, T., Momose, M., Kitamura, Y., et al. 2019, ApJ, 871, 5, doi: [10.3847/1538-4357/aaf4f1](https://doi.org/10.3847/1538-4357/aaf4f1)
- Tsukamoto, Y., Iwasaki, K., Okuzumi, S., Machida, M. N., & Inutsuka, S. 2015a, MNRAS, 452, 278, doi: [10.1093/mnras/stv1290](https://doi.org/10.1093/mnras/stv1290)

- . 2015b, *ApJ*, 810, L26, doi: [10.1088/2041-8205/810/2/L26](https://doi.org/10.1088/2041-8205/810/2/L26)
- Ueda, T., Flock, M., & Okuzumi, S. 2019, *ApJ*, 871, 10, doi: [10.3847/1538-4357/aaf3a1](https://doi.org/10.3847/1538-4357/aaf3a1)
- Ueda, T., Kataoka, A., & Tsukagoshi, T. 2020, *ApJ*, 893, 125, doi: [10.3847/1538-4357/ab8223](https://doi.org/10.3847/1538-4357/ab8223)
- Ueda, T., Okuzumi, S., & Flock, M. 2017, *ApJ*, 843, 49, doi: [10.3847/1538-4357/aa74b5](https://doi.org/10.3847/1538-4357/aa74b5)
- Uyama, T., Muto, T., Mawet, D., et al. 2020, *AJ*, 159, 118, doi: [10.3847/1538-3881/ab7006](https://doi.org/10.3847/1538-3881/ab7006)
- van der Marel, N. 2023, *European Physical Journal Plus*, 138, 225, doi: [10.1140/epjp/s13360-022-03628-0](https://doi.org/10.1140/epjp/s13360-022-03628-0)
- van der Marel, N., van Dishoeck, E. F., Bruderer, S., et al. 2013, *Science*, 340, 1199, doi: [10.1126/science.1236770](https://doi.org/10.1126/science.1236770)
- van der Marel, N., Birnstiel, T., Garufi, A., et al. 2021, *AJ*, 161, 33, doi: [10.3847/1538-3881/abc3ba](https://doi.org/10.3847/1538-3881/abc3ba)
- Verhoeff, A. P., Min, M., Pantin, E., et al. 2011, *A&A*, 528, A91, doi: [10.1051/0004-6361/201014952](https://doi.org/10.1051/0004-6361/201014952)
- Weidenschilling, S. J. 1977, *MNRAS*, 180, 57, doi: [10.1093/mnras/180.2.57](https://doi.org/10.1093/mnras/180.2.57)
- Winston, E., Cox, N. L. J., Prusti, T., et al. 2012, *A&A*, 545, A145, doi: [10.1051/0004-6361/201118665](https://doi.org/10.1051/0004-6361/201118665)
- Wright, E. L., Eisenhardt, P. R. M., Mainzer, A. K., et al. 2010, *AJ*, 140, 1868, doi: [10.1088/0004-6256/140/6/1868](https://doi.org/10.1088/0004-6256/140/6/1868)
- Zhang, K., Isella, A., Carpenter, J. M., & Blake, G. A. 2014, *ApJ*, 791, 42, doi: [10.1088/0004-637X/791/1/42](https://doi.org/10.1088/0004-637X/791/1/42)
- Zhang, K., Schwarz, K. R., & Bergin, E. A. 2020, *ApJ*, 891, L17, doi: [10.3847/2041-8213/ab7823](https://doi.org/10.3847/2041-8213/ab7823)
- Ziampras, A., Dullemond, C. P., Birnstiel, T., Benisty, M., & Nelson, R. P. 2024, arXiv e-prints, arXiv:2410.13932, doi: [10.48550/arXiv.2410.13932](https://doi.org/10.48550/arXiv.2410.13932)